A ROBUST DESIGN APPROACH TO SELECT GENETIC ALGORITHM PARAMETERS

by

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Ajit Kumar Jha

to the

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May 1997

CERTIFICATE

This is to certify that the work contained in the thesis entitled *A Robust Design Approach to Select Genetic Algorithm Parameters* by Ajit Kumar Jha, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Date: 21st May 1997

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Ajit Kumar Jha

ABSTRACT

Genetic algorithm (GA) is one of the widely used optimization methods. The efficiency of GA and its effectiveness depend to a great extent on the selection of its control parameters. One of the widely studied performance criteria for GAs is the on-line performance measure which largely depends on parameters such as population size, crossover probability, mutation probability, selection operator and crossover operator.

In the present study, variation reduction of the on-line performance measure of a simple genetic algorithm is done by the response surface methodology. The results on four different test functions suggest that proportionate selection with uniform crossover always gives consistent on-line performance of GA. For every function, the population size, crossover probability and mutation probability are found based on minimizing the variation in on-line performance measure. Since no unique values of these parameters are found to be optimal in all functions, it is not possible to conclude that a particular value of population size, crossover probability and mutation probability will always produce minimum variation in the on-line performance. However in each case the relative importance of these parameters are also found. It is observed that for a unimodal function, mutation is an important operator if used with proportionate selection and uniform crossover. For a function having a flat optimal basin, a smaller population with proportionate selection is the best. For a multimodal function, a large population with a large crossover probability is needed. This study suggests that before trying to solve a problem using GA, the procedure presented in this thesis may be applied to get consistent performance of GAs, by making the obtained solutions independent of initial population. This thesis has also outlined a methodology to obtain GA parameters which will produce consistent performance.

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Chapter 1

Introduction

The performance of a genetic algorithm (GA) depends on its parameters such as population size, genetic operator, operator probability and others. Often the performance depends on the chosen initial population. Although most theoretical studies use Markov chain modeling of the GA process, they provide little guidance for the proper selection of above parameters. Given this, it is surprising that only few studies have been done in this area. Realizing that there is a need for a more thorough investigation, in the present study optimization and variation reduction of on-line performance of the GA is done based on methods suggested by Taylor[3]. But before we discuss the present work, let us review the past efforts in designing GAs.

De Jong set out to compare GAs with the best gradient search techniques. In that study he investigated the effect of population size, crossover probability and mutation probability on on-line performance and off-line performance of GA on a number of test functions (De Jong, 1975). The following was his conclusion:

> Increasing the population size was shown to reduce the stochastic effects [of random sampling on a finite population] and improve long term performance at the expense of slower initial response. Increasing mutation rate was seen to improve off-line performance at the expense of on-line performance. Reducing the crossover rate

> resulted in an overall improvement in performance, suggesting that producing a generation of completely new individual was too high a sampling rate [5].

Based on his limited study he suggested the following ranges of GA parameters for a good on-line performance of GA:

Population size	50-100
Crossover rate	0.6
Mutation rate	0.001

De Jong also investigated the effect of multi-point crossover on his test suite. He observed that a GA with multiple cross sites exhibits different schema disruption behavior, but no significant empirical difference was observed due to different crossover sites.

A more robust approach was suggested by Grefenstette (1986) when he used a GA to address the control parameter problem by treating it as a meta-problem. That is, a meta-GA was used to locate parameter sets which themselves were used for GA searches on a set of tasks (the De Jong suite). His recommended values were [5]:

Population size	30
Crossover rate	0.95
Mutation rate	0.01

It is interesting to note that he recommended a smaller population size and much higher crossover and mutation rates than did De Jong.

Goldberg (1985) did theoretical investigations to find an optimal population size for a particular problem. Based on his simple probabilistic model of schema processing he proposed

$$pop = 1.65 \times 2^{0.2 \times length} \tag{1.1}$$

that suggests population sizes of 130, 557, 2389 and 10244 for strings of length 30, 40, 50 and 60 respectively. It is clear that this method suggests very large population for problems with moderately long chromosomes. It is also surprising that population size always depends on the string length, and not on the complexity of the problem. Realizing this, Goldberg, Deb and Clark(1992) devised a different population sizing equation based on signal to noise ratio of schema evaluation.

Robertson's (1988) investigation of population size on parallel machines found that GA's performance monotonically increased with population size. But in serial machines or other settings where there is a fixed cost increment for each population member; we do not expect the performance to continuously increase with the population size [5].

Shaffer, Caruana, Eshelman and Das [5] did a complete factorial design experiments and analysis of variance (ANOVA) was used to identify and quantify the influence of these control parameters. He concluded the following ranges of parameters for good on-line performance:

Population size	20-30
Crossover rate	0.75-0.95
Mutation rate	0.005-0.01

He suggested two point crossovers no worse and sometimes better than one point crossover.

In the all the above study the aim was to find optimum on-line performance. In none of the approach variation reduction of on-line performance was done due to change in the initial population and also no one took into account the effect of interaction between these parameters.

Bagchi and Deb [8] used design of experiment approach to calibrate the GA parameters. One of their important conclusions was that the non-linear interaction among GA parameters exist in a GA simulation. Hence the parameters cannot be optimized by studying them one at a time. For their suite of problems, they suggested to use any one of the following

combination of GA parameters :

Population size = $0.7 \times stringlength$, crossover probability = 0.75, mutation probability = 3/string length.

Population size =$1.3 \times stringlength$, crossover probability = 0.95, mutation probability = 3/string length.

Population size =$2.7 \times stringlength$, crossover probability = 0.5, mutation probability = 1/string length.

Further for problems in which equality constraints are handled by using penalty function, the following combination seemed to work well:

Population size =$1.3 \times stringlength$, crossover probability = 0.6-0.8, mutation probability < 1/string length.

The difficulty of the design of experiment approach is that it will give the optimum performance at one of the experimental settings. So one can not say that suggested combination of GA parameters will give global optimum performance.

In this study, we not only investigate the effect of GA parameters on the on-line performance, but also find optimal settings of parameters, so that the performance of GA is less sensitive to the initial population. For the GA to be a successful tool in science and engineering application, a GA must provide similar yet optimal solution with different initial populations. We use the concept of variation reduction method of robust design (Taylor, 1991), to find optimal GA parameters and show the robustness of the GA by showing results on three functions and one engineering design problem.

Chapter 2 describes the variation reduction technique. Chapter 3 outlines GA procedure in brief, and how the variation reduction technique is applied for designing GA. Chapter 4 shows the results on three functions and one engineering design problem. Chapter 5 gives a number of conclusions of the study.

Chapter 2

Variation Reduction Technique

2.1 Cause of Variation

A system input/output model can be described by Figure 2.1.

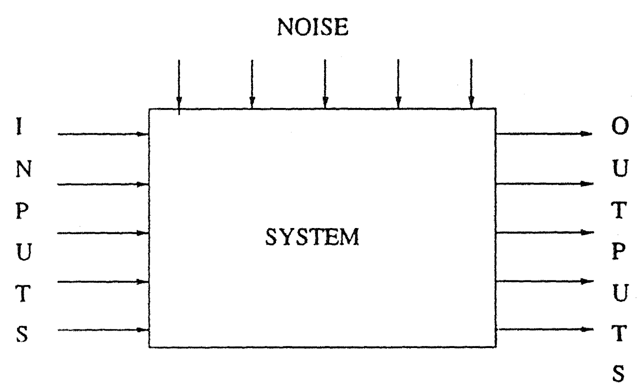

Figure 2.1: System input/output model

The system produces certain output, be they product, invoices, sales and so forth. In producing these output the system requires certain inputs like materials and workforce in manufacturing. The system may also be affected by other inputs such as environment which is called noise. The system acts on the inputs to try to achieve the desired output.

The system input/output model implies that there is some sort of relationship between the inputs and outputs. Given this relationship, often in the form of an equation or plot, the

variation transmitted to an output variable by an input variable can be determined. Figure 2.2 shows how this might be done.

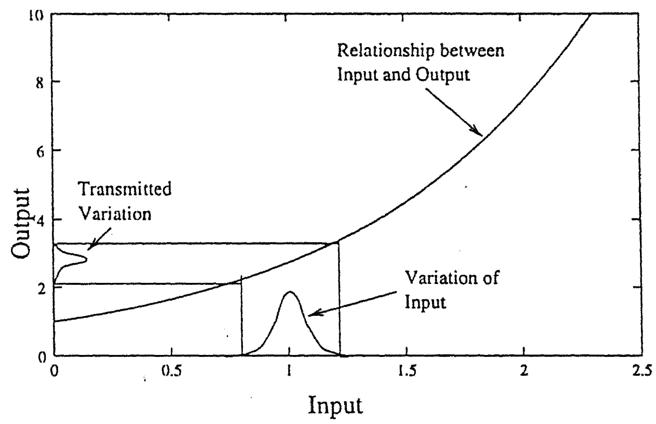

Figure 2.2: Transmission of variation

There are many inputs that might affect the system. Those inputs are called *candidate input variables.* Out of these candidate input variables, not every variable will actually have an affect on the product or process. So those inputs that can affect the output, either by affecting the average or by contributing to the variation in the output are called *key input variable.*

An important part of reducing variation is the identification of those key input variables that make significant contribution to the variation. The variation in the output is resulted by two types:-

1. Targeting the key input variable may produce variation in the output. These variables are called *variation adjustment parameters (VAP).*

2. Variation given to the key input variables may also result in variation of output. These key input variables are called *variation inducing parameter (VIP)* .

Optimization and variation reduction requires to find key input variables, VIPs and VAPs. Studying the input's effect is one of the approaches for identifying key input variables, VIPs and VAPs.

2.2 Studying the Input's Effect

When studying the input's effects, one wants to identify the key input variables, understand their effects on the average and use this understanding to optimize the average. However, one also wants to study the input's effect in order to reduce variation. There are two different approaches for accomplishing this task:

1. **Transmission of variation :** The variation transmitted by the inputs to the output can be calculated based on the relationship between the inputs and output. This relation of variation is used to find VAPs and VIPs. Section 2.2.2 deals with this method.

2. **Direct approach :** The effect that changes of the inputs have on the variation of the output is observed directly. The equation of variation is found out by fitting the best surface through the log of standard deviation. This equation of variation is used to find VAPs. Section 2.2.3 deals with this method.

The detection of key input variables is done by the screening experiment. Taylor (1995) has described in detail about the procedure for designing and analyzing screening experiments. The limitation of screening experiment is that it cannot calculate the non-linear effect of inputs, because it uses only two levels of input. Although screening experiment with some more trials including center points of the inputs can be used for calculating non-linear effects. The advantage of screening experiment is that large number (upto 16) of inputs can be analyzed with fewer number of trials.

If the number of trials is less then full factorial design of experiment can be done. Here for 3 inputs, 27 trials experiment is proposed. This 27-trial experiment is analyzed in the same way as Taylor (1995) proposed, and will be used for designing GA.

Table 2.1 provides the information required to determine where to set each input on each of the 27 trials. For each trial 5 to 10 runs are taken.

Trial number	column					
	a	b	c	d	e	f
1	2	1	2	1	2	1
2	2	2	1	2	1	1
3	1	1	2	2	1	1
4	2	1	1	1	1	2
5	1	2	1	1	2	1
6	1	2	2	1	1	2
7	2	2	2	2	2	2
8	1	1	1	2	2	2
9	3	3	3	0	0	0
10	3	1	1	0	0	2
11	3	1	2	0	0	1
12	3	2	1	0	0	1
13	3	2	2	0	0	2
14	1	3	1	0	2	0
15	1	3	2	0	1	0
16	2	3	1	0	1	0
17	2	3	2	0	2	0
18	1	1	3	2	0	0
19	1	2	3	1	0	0
20	2	1	3	1	0	0
21	2	2	3	2	0	0
22	3	3	1	0	0	0
23	3	3	2	0	0	0
24	1	3	3	0	0	0
25	2	3	3	0	0	0
26	3	1	3	0	0	0
27	3	2	3	0	0	0
	A	B	C	A*B	A*C	B*C
	Input variables corresponding to column					

Table 2.1: 27-trial experiment

First three columns are used for each of the three inputs. The columns corresponding to the inputs contain 1, 2 and 3. 2 means to set the input to its high value, 1 means to set the input to its low value and 3 means to set the input to its center value. Fourth, fifth and sixth column shows the interaction A*B, A*C and B*C respectively. 0, 1 and 2 in these columns are levels of interaction. The way to calculate level of column corresponding to (A*B) is :

Put 1 if level of parameter A and parameter B is not same i.e. A is at level 1 and B is at level 2 or vice versa. Put 2 if level of parameter A and parameter B is same i.e. A is at level 1 and B is at level 1 or A is at level 2 and B is also at level 2. Put 0 if center level 3 is present in any of parameter A or B.

For each trial average and standard deviation of the output is calculated.

Analyzing the experiment

Analyzing the experiment consists of calculating the effect of each parameter, non-liner effect of each parameter and the group of interactions, sorting the effects in order of importance,

and deciding which effects are significant.

Effect on average of individual parameter = average of trials at level 2 − average of trials at level 1

Effect on average of interaction = average of trials at level 2 − average of trials at level 1

Non-linear Effect on average = average of trials at level 1 and 2 − average of trials at level 3.

Thus by analyzing the experiment, we have the effect of each parameter, interaction and non-linear effect of each parameter on the system's performance. Out of these, the parameter with small effects are neglected.The remaining parameters are the *key input variables.*

Similar to the effect on average, effect on standard deviation of each parameter, their interactions and their non-linear effect are also found. The parameter having smaller effects are neglected. The remaining parameter which affects standard deviation are variation adjustment parameters.

2.2.1 Response Surface Studies

With all the data collected above, and with the terms having larger effect, bestfit surface is drawn through the average.

The equation for the bestfit surface will be of the form

$$\begin{aligned} Y &= a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_1^2 + a_5X_2^2 + a_6X_3^2 + a_7X_1X_2 + a_8X_2X_3 \\ &\quad + a_9X_1X_3 \qquad (2.1) \end{aligned}$$

Where Y is the output, X_i represents the input, and a_i is a constant to be determined based on the data.

In the above polynomial equation only 2^{nd} order terms are used. The terms with higher order can be included but then complexity increases. Hence for simplicity 2^{nd} order

polynomial is used.

Bestfit surface through log of standard deviation is also found out, with the parameters affecting the standard deviation.

The analysis described above shows how to find key input variable, what will be the terms in bestfit surface through average and what is the bestfit equation through the average.

The analysis also shows how to find variation adjustment parameters , what will be the terms in bestfit surface through log of standard deviation and what will be equation through log of standard deviation.

2.2.2 Variation Transmission Analysis

The bestfit surface for the average will be of the form

$Y = g(X_1, X_2, X_3, ..., X_n)$

Let us use the following notations:

$g_1(X_1, X_2, X_3, ..., X_n)$ = first partial with respect to X_1

$g_2(X_1, X_2, X_3, ..., X_n)$ = first partial with respect to X_2

. . . . . .

$g_n(X_1, X_2, X_3, ..., X_n)$ = first partial with respect to X_n.

$g_{1,1}(X_1, X_2, X_3, ..., X_n)$ = Second partial with respect to X_1

$g_{2,2}(X_1, X_2, X_3, ..., X_n)$ = Second partial with respect to X_2

. . . . . .

$g_{n,n}(X_1, X_2, X_3, ..., X_n)$ = Second partial with respect to X_n

$g_{1,2}(X_1, X_2, X_3, ..., X_n)$ = Second partial with respect to X_1 and X_2

$g_{1,3}(X_1, X_2, X_3, ..., X_n)$ = Second partial with respect to X_1 and X_3

...

$g_{n-1,n}(X_1, X_2, X_3, ..., X_n)$ = Second partial with respect to X_{n-1} and X_n

Then the squared standard deviation is approximately equal to (Taylor, 1991)

$$\begin{aligned}\sigma_y^2 \quad = \quad & g_1(t_1, t_2, ..., t_n)^2\sigma_1^2 + g_2(t_1, t_2, ..., t_n)^2\sigma_2^2 + ... + g_n(t_1, t_2, ..., t_n)^2\sigma_n^2 + \\ & 0.5 \times g_{1,1}(t_1, t_2, ..., t_n)^2\sigma_1^4 + 0.5 \times g_{2,2}(t_1, t_2, ..., t_n)^2\sigma_2^4 + \\ & .... + 0.5 \times g_{n,n}(t_1, t_2, ..., t_n)^2\sigma_n^4 + g_{1,2}(t_1, t_2, ..., t_n)^2\sigma_1^2\sigma_2^2 + \\ & g_{1,3}(t_1, t_2, ..., t_n)^2\sigma_1^2\sigma_3^2 + .. + g_{n-1,n}(t_1, t_2, ..., t_n)^2\sigma_{n-1}^2\sigma_n^2 \end{aligned} \tag{2.2}$$

Where $t_1, t_2...$, are the value of key input parameter and $\sigma_1, \sigma_2...$, are the standard deviations of the key input parameter. In the above equation $\sigma_1, \sigma 2..$ are known because we know the variation of the key input variable. The above equation is minimized for reducing the variation. The optimum value of $t_1, t_2..$ will be required at which we have to target that parameter (This is called the parameter design) for reducing the variation. These parameters are variation adjustment parameters (VAPs).

Tolerance Design

The purpose of tolerance design is to select tolerance for each of the inputs around the target selected during parameter design. As a part of the parameter design, an equation for the standard deviation of the output is derived. This equation expresses the standard deviation of the output as a function of the input's target $t_i's$ and the input's standard deviation $\sigma_i's$.

$$\sigma_y = f(t_1, t_2, ..., t_n, \sigma_1, \sigma_2, .., \sigma_n) \tag{2.3}$$

Putting the value of $t_i's$ into this equation simplifies the equation so that σ_y is simply function of the input's standard deviation.

$$\sigma_y = f(\sigma_1, \sigma_2, .., \sigma_n) \tag{2.4}$$

The transmitted contribution of an individual input σ_{ti} is

$$\sigma_{t1} = \sqrt{f(\sigma_1, \sigma_2, .., \sigma_n)^2 - f(0, \sigma_2, .., \sigma_n)^2}$$

$$\sigma_{t2} = \sqrt{f(\sigma_1, \sigma_2, .., \sigma_n)^2 - f(\sigma_1, 0, .., \sigma_n)^2} \tag{2.5}$$

$$....$$

$$\sigma_{tn} = \sqrt{f(\sigma_1, \sigma_2, .., \sigma_n)^2 - f(\sigma_1, \sigma_2, .., 0)^2}$$

These standard deviation of the inputs transmitted variation can be ranked. Those inputs with largest $\sigma_{ti}'s$ are the variation inducing parameters (VIPs).

The tolerance on these VIPs are tightened to reduce the variation. Those key input variables which are not VIPs are found out from the bestfit surface for average, such that average is minimized.

The procedure of the variation transmission analysis can be summarized as:

1. 27-trial experiment is done. The analysis of experiment is done as described in previous section. Then response surface study is done.

2. The bestfit equation for average is obtained from response surface studies, equation 2.1.

3. The equation for square of standard deviation can be obtained from equation 2.2.

4. By putting the standard deviation of each input in equation 2.2 and minimizing it we will get the value of parameter t_i . This t_i is the value at which the corresponding parameter should be targeted. These parameters are called (VAPs).

5. These $t_i's$ are put into equation 2.2, which results in equation 2.4. Equation 2.5 is used to calculate transmitted variation of each input. The parameter having large transmitted variation are VIPs.

2.2.3 Direct Approach

The analysis involves analyzing the standard deviation just as one would analyze the average. Procedure for the direct approach is as follows:

1. 27-trial experiment is done. The analysis of experiment is done as described in previous section. Then response surface study is done.

2 The bestfit equation for the standard deviation is obtained from response surface studies.

3 The equation for bestfit surface through standard deviation is minimized, and the value of the parameters for minimum standard deviation are found out. These parameters are VAPs. The parameters has to be varied around this value of VAPs.

4 The value of VAPs is put into equation for average. The resulting equation is minimized. The value of the parameter obtained after minimization is to where, we have to set it for optimization of average.

The following is the limitation of the *direct approach*

1. Since only few samples are taken for each experiment, standard deviation calculated may not represent actual standard deviation. In that case some of the VAPs may be missed.

2. The direct approach identifies VAPs and helps to target them to minimize the variation. The direct approach can't isolate the contribution of individual inputs required for tolerance design. Thus it can't find VIPs.

2.2.4 Comparison between Direct Approach and Variation Transmission Analysis

Despite limitation in direct approach method, using this method is certainly worthwhile. As part of any analysis the standard deviation should be analyzed as well as the average. The

added information is essentially free, as no additional data are required. Any VAPs that are identified can result in valuable improvements.

However one should not rely solely on direct observation. It is still worthwhile to perform a variation transmission analysis. Frequently, VAPs are identified that are missed by direct observation. Further, the variation transmission analysis can be used to perform tolerance design including identifying VIPs.

Let's turn to example to see the difference exactly. Suppose we assume input output relation as:

$$Y = 8.125 - 0.4X + 0.185X^2 - 0.006X^3 \tag{2.6}$$

A plot of the effect of X on Y is shown in Figure 2.3.

The variation of Y is approximately

$$\sigma_y = \sqrt{(-0.4 + 0.37t_x - 0.018t_x^2)^2\sigma_x^2 + 0.5(0.37 - 0.036t_x)^2\sigma_x^4} \tag{2.7}$$

Further assume that $\sigma_x = 0.2$. A plot of the effect of t_x on σ_y is shown in Figure 2.4. The standard deviation of Y is 0.2001 when $t_x = 5$, 0.2725 when t_x is 7.5, and 0.3 when $t_x = 10$.

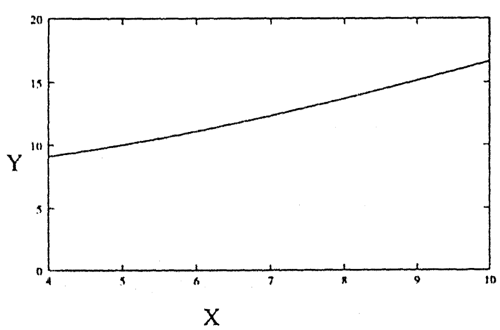

Figure 2.3: Effect of input X on output Y

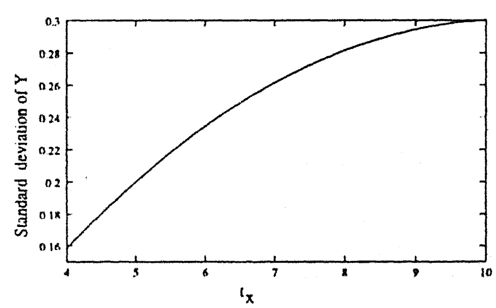

Figure 2.4: Effect of t_X on σ_Y

The sizes of the true linear and quadratic effects are

True linear effect $= 0.3 - 0.2001 = 0.0999$

True quadratic effect $= 0.2725 - (0.2001 + 0.3)/2 = 0.02245$

Estimation by Direct Observation

One way of doing this would be to take ten samples when $t_x = 5$,ten more samples when $t_x = 7.5$, and a final set of ten samples when $t_x = 10$. For each group of ten samples, the standard deviations could be calculated and used to estimate the effect of t_x on σ_y . One such data set is given in Table 2.2.

	Target of X		
	5	7.5	10
Values	10.36	13.40	16.55
	10.07	13.43	16.91
	10.14	13.17	16.92
	10.28	13.28	16.64
	9.71	13.34	16.67
	10.19	13.30	16.55
	10.05	13.08	16.68
	9.98	13.13	17.10
	10.37	12.57	17.04
	9.91	12.78	16.74
Average	10.11	13.15	16.78
Std. Dev.	0.208	0.278	0.199

Table 2.2: Data for estimating effect of X on variation of Y

The observed linear and quadratic effects of adjusting t_x are

Observed linear effect $= 0.199 - 0.208 = -0.009$

Observed quadratic effect $= 0.278 - (0.199 + 0.208)/2 = 0.0745$

A plot of standard deviation is shown in Figure 2.5 along with the true values. This demonstrates the inaccuracy involved in estimating standard deviation with small sample sizes.

Estimation by Transmission of Variation

The bestfit curve to the data of average in Table 2.2 is

$$Y = 5.7755 + 0.6318X + 0.046X^2 \tag{2.8}$$

The best fit curve is shown in Figure 2.6 along with the true curve.

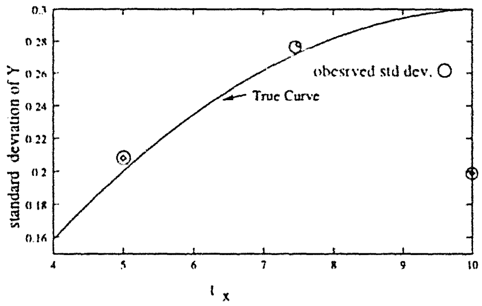

Figure 2.5: Observed standard deviations

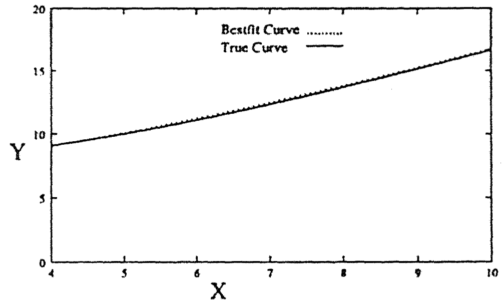

Figure 2.6: Observed averages

Using the above equation the standard deviation of Y can be estimated as follows

$$\sigma_y = \sqrt{(0.6318 + 0.092t_x)^2\sigma_x^2 + 0.5(0.092)^2\sigma_x^4} \tag{2.9}$$

The standard deviation of X was found to be 0.1704.

Evaluating the estimate of σ_y gives 0.1875 at $t_x = 5$, 0.2274 at $t_x = 7.5$, and 0.2674 at $t_x = 10$.

Using these result the predicted linear and quadratic effects can be calculated as follows:

predicted linear effect $= 0.2674 - 0.1875 = 0.0799$

predicted quadratic effect $= 0.2274 - (0.2674 + 0.18750)/2 = -0.00005$

The predicted effect of t_x on σ_y is shown in Figure 2.7.

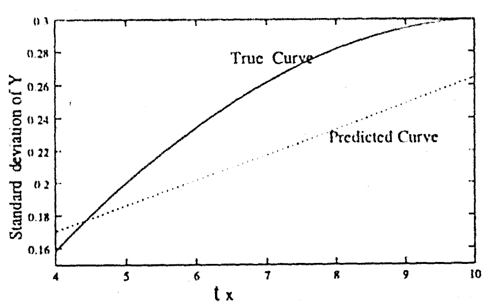

Figure 2.7: Predicted standard deviations

Once again there is lack of fit due to variation in the data and the fact that the true cubic polynomial is approximated by a quadratic polynomial.

By the above example we saw the limitation of both the method. In the practical case, when we are applying this method to reduce the variation, the bestfit surface through average and through standard deviation may not represent the relation between input and output correctly. In that case both the direct method and transmission of variation method are applied.

Chapter 3

Designing A Genetic Algorithm

3.1 Introduction to Genetic Algorithm

Genetic Algorithm (GA) is a search procedure based on the mechanics of natural selection and natural genetics. It uses the principle of survival of fittest same as nature. It abstracts operators from nature to form a mechanism that is suitable for a variety of search problems.

In a simple GA, variables are coded in the form of strings containing *alleles* (either 1 or 0) similar to the *chromosomes* having genes in a biological system. In GA's terminology, strings along with their decoded value are known as *individuals*. A set of such individuals in the user-set range is called a *population*. Like in the natural genetics, GA processes different population through generation. The three basic operators that modify the population to another are as follows:

- Reproduction
- Crossover
- Mutation

The first operation employed in GA is reproduction or selection. In the population each individual is evaluated to find its fitness value. The fitness value is same as function value in

the case of maximization else in minimization it is $\frac{1}{1+f}$. In the selection process, individuals are given their presentation or copies according to their fitness value in order to form a set of good individuals called *mating pool*.

One of the method of selection is the proportionate reproduction in which a string is selected for the mating pool with a probability proportional to its fitness value.

Another method of selection is the tournament selection in which s individuals (with or without replacement) from the population are picked and best one is selected for the mating pool. In most GA applications, a binary tournament selection with $s = 2$ is used.

The second operation employed in a GA is crossover. In a single point crossover, among the individuals in mating pool two are picked at random and a random cross site is selected across the string length. Alleles on one side are swapped between the individuals to form two different strings called the children points. For example, if s_1 and s_2 are two individuals and a random cross site is selected as shown:

```
s1 = 1 1 0| 1 0 1
s2 = 1 0 1| 0 1 0
```

After mating the two children points s_1' and s_2' will be

```
s1' = 1 1 0 0 1 0
s2' = 1 0 1 1 0 1
```

In the uniform crossover bit by bit crossover is done with a probability 0.5. After mating the two children points s_1' and s_2' are

$$\begin{aligned} s1' &= 1 \ \boxed{0} \ 0 \ \boxed{0} \ 0 \ 1 \\ s2' &= 1 \ \boxed{1} \ 1 \ \boxed{1} \ 1 \ 0 \end{aligned}$$

Crossover is the only operator which creates new individuals by mixing the genetic information of parent individuals. This operation is done with a probability known as the crossover

probability p_c.

Mutation, the third operator, plays a secondary role in a GA. This is applied to get some features which cannot be achieved by crossover. Mutation is occasional alteration of an allele (changing 0 to 1 and vice versa) with some probability called mutation probability p_m. The action of mutation operator is shown below:

$$1\ 0\ 1\ \boxed{0}\ 1 \longrightarrow \quad 1\ 0\ 1\ \boxed{1}\ 1$$

A flow chart of simple GA is shown in Figure 3.1.

If proper GA parameters are not used, the performance of GA may change as the initial population is changed. In order to get consistent result, a variation reduction study is needed, which will give the approximate range of parameters for a consistent performance of GA. To quantify the effectiveness of different genetic algorithms, De Jong devised two performance measures, one to gauge convergence and other to gauge ongoing performance. He called these measure off-line (convergence) and on-line (ongoing) performance respectively.

On-line performance is an average of all function evaluations upto and including current trial (De Jong, 1975). In a minimization problem if on-line performance is less that means that the performance of GA is good.

3.2 Application of Variation Reduction Technique for Designing GA

The first step is to decide the performance measure. In the present study, the on-line performance of last 50 % generations is chosen as the performance measure. The various parameters that can affect on-line performance are :

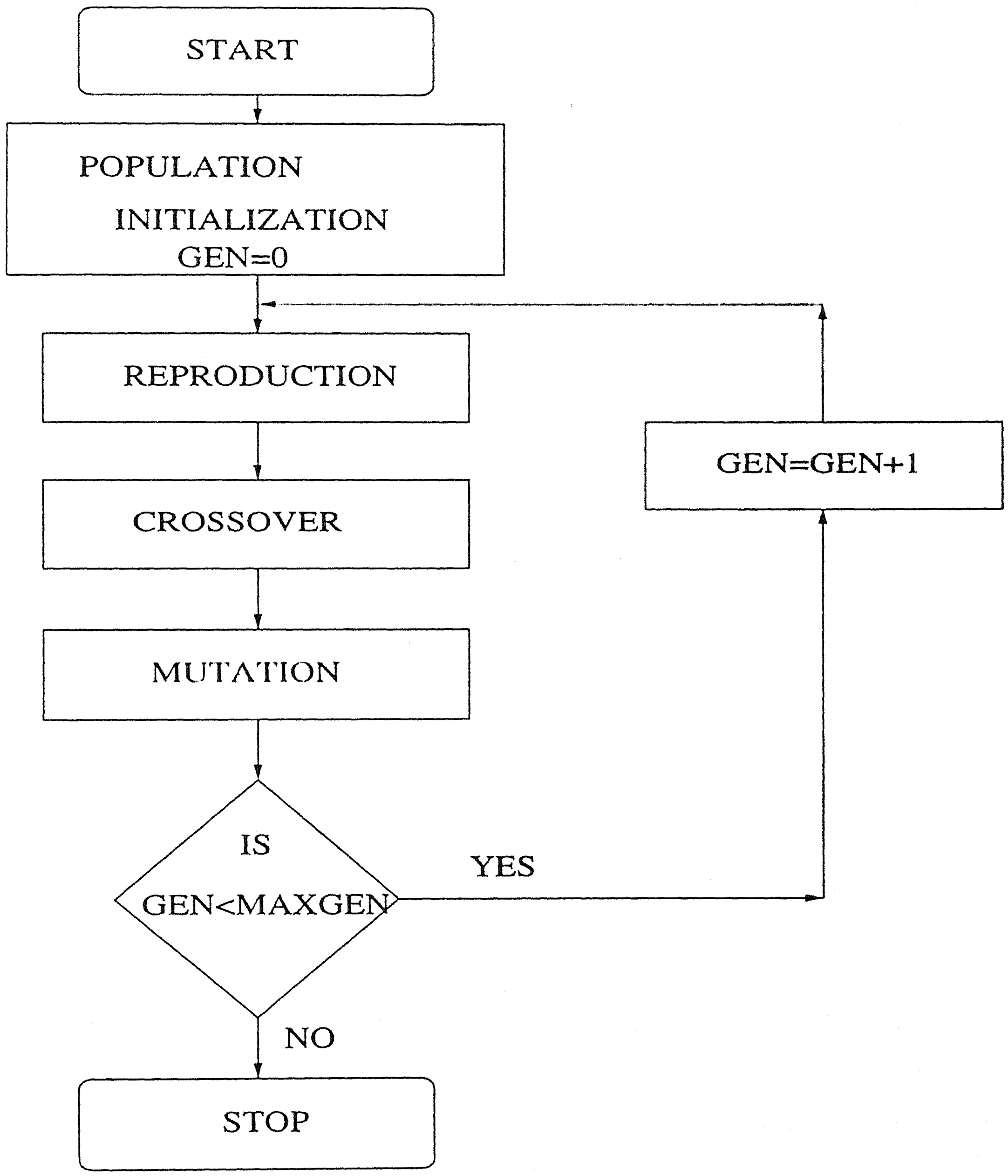

Figure 3.1: Flow chart of simple Genetic Algorithm

1 String length

2 Cardinality of alphabet i.e. binary, ternary, real coded etc.

3 Linkage i.e. tight coding, loose coding, random coding.

4 Population size

5 Crossover probability

6 Mutation probability

7 Selection operator i.e. proportionate selection, ranking selection, tournament selection, genitor selection.

8 Crossover operator i.e. single point crossover, two point crossover, uniform crossover.

9 Mutation operator i.e. bitwise mutation, mutation clock.

10 Termination criteria

11 Sequence of GA operator

If all the inputs will be considered for designing GA, the computational complexity is large. In the present study only a few inputs are fixed to reasonable values and others are varied. In the present study

- Binary GA is used.
- String length is not taken as input parameter as affecting output, because string length mainly affects accuracy.
- Tight coding is used as the linkage parameter.
- Out of the various selection procedures tournament selection and proportionate (rowlett wheel) selection is chosen as input affecting on-line performance.

- Out of the various crossover operators single point crossover and uniform crossover is chosen as input affecting on-line performance.
- Bitwise mutation is used as the mutation operator.
- Ten thousand function evaluation is chosen as termination criteria.
- Reproduction then crossover then mutation is chosen as sequence of GA operator.
- Simple GA written in 'C' programming language is used.

Following are the parameters which is considered as candidate input variable:

1 Population size (PS) in the range $0.25 \times stringlength$ to $2.5 \times stringlength$

2 Crossover probability (CP) in the range 0.5 to 1.0

3 Mutation probability (MP) in the range 0.001 to 0.1

4 Selection operator (SP)

5 Crossover operator (CO)

Out of the above parameter key input variable, VAPs and VIPs are to be found. Since selection procedure and crossover operator are discrete variable, handling this is found to be difficult when bestfit surface through average and standard deviation is found. So for each of the test function study, four cases are made and in each case these parameters is fixed.

Case 1 Proportionate selection, single point crossover is used.

Case 2 Proportionate selection, uniform crossover is used.

Case 3 Tournament selection, single point crossover is used.

Case 4 Tournament selection, uniform crossover is used.

For each of the cases population size, crossover probability and mutation probability are used as candidate input variable.

In the present case following test functions is studied for optimization and variation reduction of on-line performance.

1 DeJong F1 :

Minimize $f(x) = \sum_{i=1}^{3} x_i^2$

$$-5.12 \leq x_i \leq 5.12$$

This is a three variable unimodal function having a minima at $(0,0,0)^T$. The GA will find the minimum point easily. The purpose of taking this function is to reduce the variation in the on-line performance of one of the simplest problem. That is why variation reduction is first tried on this function.

2 DeJong F2 :

Minimize $f(x) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$

$$-2.048 \leq x_i \leq 2.048$$

This is a two-dimensional minimization problem with the global minimum at $(1,1)^T$. In this function, the slope towards the optimum point is very small and the population gets converged to a point along the ridge. The idea behind using this function is to apply variation reduction on slightly difficult function.

3 Rastrigin's function:

Minimize $f(x) = 200 + \sum_{i=1}^{20}(x_i^2 - 10\cos(2\Pi x_i))$

$$-5.12 \leq x_i \leq 5.12$$

This function has 20 variables and contains 2^{20} local minimum points, of which only one is the global minimum point. The global minimum point is at $x_i = 0$ for all

$i = 1, 2, \ldots, 20$. At this point the function value is zero. The other local minima occur when the cosine term is one. This function is a difficult optimization problem, because there are 2^{20} different local optimal points (surrounding the global optimum) with function values very close to that at the global optimal point. This function is used to show the variation reduction on one of difficult optimization problem.

4 welded beam problem:

For showing the variation reduction on one of engineering design problem, welded beam problem is selected.

Minimize cost $= 1.1x_1^2x_2 + .048x_3x_4(L + x_2)$

subjected to

$g_1(x) = S_{yt} - \sigma(x) \geq 0$

$g_2(x) = S_{sy} - \tau(x) \geq 0$

$g_3(x) = P_c(x) - F \geq 0$

$g_4(x) = x_4 - x_1 \geq 0$

The welded beam is shown in Figure 3.2.

The parameter L is the length of the beam not welded and this is kept fixed to 14 inches. The cost of fabrication includes material cost and the welding labour cost. The first constraint limits the bending stress ($\sigma(x)$) anywhere in the welded beam to the maximum allowable strength (S_{yt}) of the beam material. The second limits the shear stress ($\tau(x)$) in the weld to the allowable shear stress (S_{sy}) of the material. The third constraint restricts the buckling of the plate ($P_c(x)$) due to applied load. The fourth constraint makes sure that the weld size (h) is less than the thickness (b) of the beam. The details of different stresses and loads are given by:

Bending stress, $\sigma(x) = \frac{6FL}{(x_4x_3^2)}$

Shear stress is given in terms of primary stress (τ') and secondary stress (τ'') as:

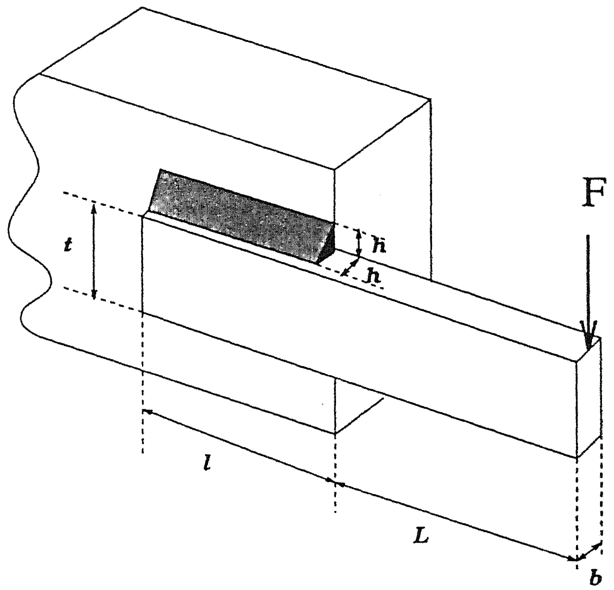

Figure 3.2: Diagram of welded beam problem

$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau'' \cos(\theta) + (\tau'')^2}$

where

$\tau' = \frac{F}{\sqrt{2}x_1x_2}$,

$\tau'' = \frac{MR}{J}$,

$M = F(L + \frac{x_2}{2})$,

$R = \sqrt{\frac{x_2^2}{4} + (\frac{x_3+x_1}{2})^2}$

$J = 1.414x_1x_2(\frac{x_2^2}{12} + (\frac{x_3+x_1}{2})^2)$

The parameter M is the moment of F about the center of gravity and J is the polar moment of inertia of the weld group and $\cos\theta = \frac{x_2}{2R}$.

Buckling load is given by :

$P_c(x) = \frac{4.013\sqrt{EI\alpha}}{L^2}(1 - \frac{x_3}{2L}\sqrt{\frac{EI}{\alpha}})$

where moment of inertia $I = \frac{1}{12}x_3x_4^3$, $\alpha = \frac{1}{2}Gx_3x_4^3$,

$E = 30 \times 10^6 psi$ and shear modulus $G = 12 \times 10^6 psi$. The allowable shear strength taken is $S_{yt} = 30000$ psi and maximum shear stress is $S_{sy} = 13600psi$. The load F is 6000 lb.

Chapter 4

Results

4.1 Test Function 1: De Jong Function F1

4.1.1 Initial Capability Study

In order to emphasize the need for a variation reduction study a capability study is first performed. In the capability study, twenty different trials are taken. For each trial all five variables are selected at random and ten runs are performed each time with a different initial population. The on-line performance measured for last 50 % of the generation are shown in Table 4.1. The average and standard deviation of all 20 trials are also shown in the table.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	30.99	31.22	32.57	31.52	31.91	31.23	31.13	31.79	31.69	31.47	31.55	0.46
2	19.39	18.06	17.80	18.88	19.51	18.14	21.50	17.95	19.87	20.66	19.18	1.25
3	4.94	00.19	00.59	5.85	00.30	00.04	00.13	00.55	00.08	00.37	01.30	2.17
4	23.47	21.77	23.43	20.86	21.40	21.08	21.99	22.79	24.12	19.95	22.09	1.33
5	20.24	11.47	02.93	01.93	06.08	03.14	21.62	21.27	23.50	17.14	12.93	8.78
6	32.01	33.72	33.32	32.45	33.26	33.45	34.93	36.09	32.34	33.15	33.47	1.23
7	24.64	23.75	24.25	23.75	24.31	23.89	24.15	23.82	23.52	24.08	24.01	0.33
8	14.57	15.39	15.95	14.38	21.27	21.39	22.19	20.61	16.14	20.29	18.22	3.17
9	66.84	66.82	67.18	67.03	65.85	65.14	66.62	66.24	65.02	66.22	66.30	0.76
10	69.70	69.00	69.92	70.01	69.69	69.56	70.10	70.24	69.34	69.75	69.84	0.27
11	43.30	42.75	40.68	42.75	41.27	41.43	41.38	41.39	41.88	40.72	41.76	0.90
12	61.77	62.05	63.09	60.73	60.92	62.26	62.64	61.14	62.16	62.27	61.90	0.76
13	71.71	71.64	71.83	72.21	72.06	71.94	71.91	71.98	71.67	72.17	71.91	0.20
14	20.55	20.33	23.14	20.38	20.92	21.82	20.47	20.95	20.32	19.59	20.85	0.99
15	37.46	36.39	36.35	37.52	37.97	37.34	38.16	37.33	37.26	35.80	37.16	0.75
16	53.81	50.44	50.41	52.19	50.20	53.20	52.47	51.73	51.59	51.66	51.77	1.20
17	22.31	21.10	22.97	24.95	22.26	23.67	22.21	22.48	23.65	24.57	23.02	1.19
18	10.92	17.49	10.41	06.38	05.46	06.89	09.40	06.84	11.53	03.91	08.92	3.93
19	34.44	33.43	35.09	34.65	34.45	36.81	34.73	33.03	33.86	37.59	34.81	1.42
20	37.28	35.60	32.48	36.09	32.83	34.31	35.58	35.08	36.38	33.97	34.96	1.55

Table 4.1: Initial capability study for function F1

The standard deviation of each trial is the measure of variation within the trial due to change in initial population. This table is used to construct the average and standard deviation *control charts* as shown in Figure 4.1 and Figure 4.2.

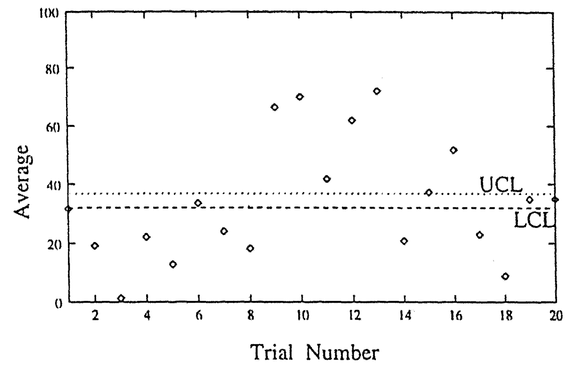

Figure 4.1: Control chart for average on-line performance before variation reduction for function F1

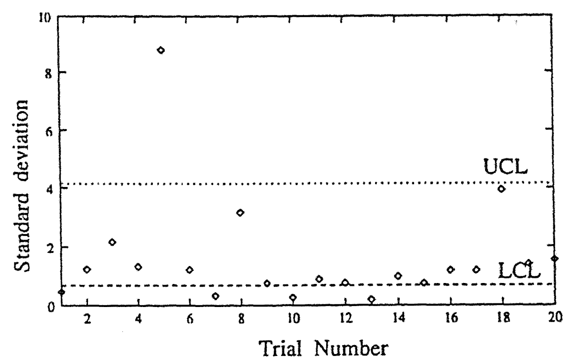

Figure 4.2: Control chart for standard deviation of on-line performance before variation reduction for function F1

If X_i be the average of every trial and S_i be the standard deviation of every trial, then the grand average is given as follows:

$$\bar{X} = \frac{\sum_{i=1}^{20} X_i}{20} \tag{4.1}$$

and the standard deviation is given as follows:

$$S = \sqrt{\frac{\sum_{i=1}^{20} S_i^2}{20}} \tag{4.2}$$

Accepting acceptable trials within $\pm 3\sigma$ from mean, the upper control limit of the average is

$$UCL_x = \bar{X} + \frac{3S}{\sqrt{n}}, \tag{4.3}$$

and the lower control limit of the average is

$$LCL_x = \bar{X} - \frac{3S}{\sqrt{n}}. \tag{4.4}$$

Similarly, the upper control limit of the standard deviation is given as

$$UCL_s = B_6 S \tag{4.5}$$

and the lower control limit of the standard deviation is given by

$$LCL_s = B_5 S \tag{4.6}$$

where $B_6 = 1.669$ and $B_5 = 0.2759$ for number of 10 runs (Taylor,1991).

For the data in Table 4.1, we compute the above quantity:

$$\begin{aligned} \bar{X} &= 34.297 \\ S &= 2.483 \\ UCL_X &= 36.65 \\ LCL_X &= 31.94 \\ UCL_S &= 4.145 \\ LCL_S &= 0.6852 \end{aligned}$$

The *average control chart* is a plot of the averages of all twenty trials. The control limit represents the extent that the group average should vary if the input variables are changed. The *standard deviation control chart* is a plot of the standard deviation of all twenty trials. Figure 4.1 shows the plot of average of on-line performance of all 20 trials. The figure clearly shows that points of the trials are not within acceptable region. Similarly Figure 4.2 shows the standard deviation of all 20 trials, of which 5 trials are outside acceptable limits.

Thus, the GA considered above is not a capable process. In order to make the GA search process capable, we need to use the variation reduction study as described in the previous chapter.

Among the five parameters, the selection operator and the crossover operator are considered only of two types each. Thus, we consider four different cases each with a particular combination of selection and crossover operator and the effect of three parameters — population size, crossover probability and mutation probability are studied.

4.1.2 CASE 1 (Proportionate Selection and Single Point Crossover)

There are only three parameters to study in this case. Thus, 27 trial experiment is performed as described in chapter 2. For each trial, with proportionate selection as selection operator and the single point crossover as the crossover operator and by choosing the combination of three parameter from Table 4.2, GA runs are performed. For each trial, ten runs are performed by changing the initial population. The average and the standard deviation of these 10 runs are calculated for each trial and are tabulated in Table 4.2.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	112	0.50	0.100	24.9	25.6	24.9	25.4	24.8	24.9	24.0	24.9	25.3	25.5	25.02	0.46
2	112	1.00	0.001	13.3	13.1	13.1	8.3	17.6	15.1	13.4	14.0	10.3	11.8	13.00	2.54
3	11	0.50	0.100	15.2	13.6	12.9	12.4	13.9	14.3	13.8	12.4	12.6	12.5	13.36	0.94
4	112	0.50	0.001	14.2	15.8	14.9	7.4	11.3	17.0	15.5	17.7	16.0	21.0	15.08	3.67
5	11	1.00	0.001	2.0	1.1	1.4	1.0	1.3	1.3	0.8	0.1	0.0	0.0	0.90	0.68
6	11	1.00	0.100	13.9	17.5	12.8	13.0	13.5	15.8	12.7	12.6	12.4	12.6	13.68	1.67
7	112	1.00	0.100	25.0	24.6	24.7	24.8	24.5	24.0	25.0	24.8	24.1	25.3	24.68	0.42
8	11	0.50	0.001	21.0	17.7	18.1	0.4	0.0	0.0	0.1	0.0	0.1	0.1	5.75	9.15
9	61	0.75	0.051	20.5	23.4	19.9	20.6	21.0	21.3	22.2	20.8	20.1	20.4	21.02	1.05
10	61	0.50	0.001	2.4	4.8	3.1	1.5	0.7	1.2	2.2	5.5	1.5	10.4	3.31	2.93
11	61	0.50	0.100	22.8	22.7	23.1	23.7	23.3	23.1	22.6	22.7	23.6	23.4	23.08	0.40
12	61	1.00	0.001	4.4	1.5	1.6	2.0	7.7	0.2	5.3	3.4	4.9	0.9	3.18	2.37
13	61	1.00	0.100	23.7	23.6	23.5	23.4	23.5	22.9	23.0	23.2	23.3	23.0	23.31	0.27
14	11	0.75	0.001	8.2	0.3	0.1	0.1	0.1	1.2	1.1	0.9	0.7	0.4	1.30	2.47
15	11	0.75	0.100	13.7	14.0	18.8	15.6	15.8	13.2	15.9	13.4	15.4	18.6	15.43	1.99
16	112	0.75	0.001	10.7	19.0	11.9	14.9	23.9	18.0	14.3	15.2	22.2	10.6	16.06	4.62
17	112	0.75	0.100	24.9	25.1	25.1	25.2	25.3	24.7	24.4	24.7	25.1	24.5	24.90	0.30
18	11	0.50	0.051	13.7	20.3	19.2	18.9	19.8	18.9	19.0	19.4	18.5	18.4	18.60	1.82
19	11	1.00	0.051	9.4	7.9	7.6	8.2	7.3	7.6	12.0	10.0	11.6	10.2	9.20	1.72
20	112	0.50	0.051	23.2	22.8	23.1	23.3	24.4	24.5	22.9	24.3	23.9	23.7	23.63	0.65
21	112	1.00	0.051	23.1	24.2	22.3	24.8	24.0	24.5	23.2	23.7	24.6	25.0	23.95	0.85
22	61	0.75	0.001	5.1	0.4	0.3	3.4	4.4	1.4	2.6	5.7	1.8	0.3	2.53	2.03
23	61	0.75	0.100	23.1	23.4	22.7	22.9	23.9	23.5	22.6	23.2	23.2	23.6	23.21	0.42
24	11	0.75	0.051	10.1	14.3	9.8	10.9	14.0	15.1	10.2	8.5	7.4	8.8	10.91	2.67
25	112	0.75	0.051	24.9	25.3	23.0	24.0	24.3	23.3	24.1	24.8	24.7	23.5	24.17	0.75
26	61	0.50	0.051	21.2	20.7	22.9	21.1	20.8	23.4	18.9	21.8	22.2	22.0	21.50	1.28
27	61	1.00	0.051	21.2	21.6	19.3	20.7	22.6	21.3	21.9	20.5	22.4	20.9	21.25	0.97

Table 4.2: Data obtained from 27 trials on DeJong F1 function case 1

The effect of each parameter and their interaction with each other is calculated. The quadratic effect of each parameter is also calculated by the same way as described in chapter 2.

The significant effects of parameters, their interactions and quadratic effect are shown in Table 4.3 for the average. The table shows that PS, CP and MP affect the average on-line performance. The significant effects are linear in PS, CP, MP and PS*CP, and quadratic in MP. These linear effects of PS, CP and MP represent the amount that average of on-line performance of the trial changes as a result of adjusting the parameter from their low value

to high value.

Design parameter		Effect
MP	mutation prob.	13.951
PS	population size	11.26
MP*MP	mutation prob. (quad)	5.593
PS*CP	population size * crossover prob.	1.9734
CP	crossover prob.	-1.796

Table 4.3: Effects table for average case 1

The effect that design parameters have on standard deviation is shown in Table 4.4 . Since MP and CP*MP have significant effect on standard deviation means MP and CP are VAPs.

Design parameter		Effect
MP	mutation prob.	2.62
CP*MP	crossover prob.* mutation prob.	1.7864

Table 4.4: Effects table for standard deviation case 1

Response surface study

We fit the best surface on the average of on-line performance from data presented in Table 4.2. The resulting equation is :

$$\begin{aligned} \text{On-line performance} &= 6.8046 + 0.059PS + 226.1MP - 7.7CP - 873.63MP^2 \\ &\quad +0.069PS \times CP \end{aligned} \tag{4.7}$$

This equation can be used to perform the variation transmission analysis. As part of variation transmission analysis the tolerances for the design parameters can be established for a capable GA.

The best fit surface through the standard deviation of on-line performance presented in Table 4.2, is also found for doing analysis by direct observation. The surface through

logarithm (due to statistical reason) of *standard deviation* is passed. The resulting equation is:

$$\log(\sigma) = 1.09 - 17.87MP + 4.66CP \times MP \tag{4.8}$$

Analysis by direct observation

We see that MP and CP are VAPs as their effects are significant on the standard deviation of the on-line performance. The minimization of the equation 4.8 gives

$$\sigma = 0.628 \text{ at } CP = 0.5 \text{ and } MP = 0.1.$$

These values of CP and MP can be put into equation 4.7 of on-line performance and on-line performance is minimized, keeping PS as the only variable. Since, in this case there is only one variable, we can minimize the above equation by hand calculation. But in case, there are more variables, GA is used for minimization.

$$\text{Minimum on-line performance} = 17.8568 \text{ at } PS = 11$$

The direct observation can't find VIPs. Based on these analysis, we conclude the following:

- Set CP = 0.5 and MP = 0.1 in order to minimize variation.
- Set PS = 11 to reduce the average.

Analysis by Variation transmission

Variation transmission analysis requires an equation such as one obtained for the on-line performance. It assumes that the average of the process parameters are t_{PS}, t_{CP}, t_{MP} and that these parameters vary around their averages with standard deviations $\sigma_{PS}, \sigma_{CP}, \sigma_{MP}$ respectively. Based on these assumptions, the standard deviation of on-line performance can be determined as follows:

$$\sigma^2 = (0.059 + 0.069t_{CP})^2\sigma_{PS}^2 + (-7.7 + 0.069t_{PS})^2\sigma_{CP}^2 +$$

$$(226.1 - 2 \times 873.63 t_{MP})^2 \sigma_{MP}^2 + 0.5 \times (2 \times 873.63)^2 \sigma_{MP}^4 + 0.069^2 \sigma_{PS}^2 \sigma_{CP}^2 \quad (4.9)$$

We see that the terms t_{PS}, t_{CP}, t_{MP} are present in the above equation of standard deviation. This means that PS, CP and MP all are VAPs. In the above equation, the following values of standard deviation in the parameters are assumed.

$$\sigma_{PS} = 5, \qquad \sigma_{CP} = 0.05, \qquad \sigma_{MP} = 0.005.$$

Putting these values in the equation 4.9 and minimizing the equation, yields:

$$\sigma = 0.53 \text{ at } PS = 105, \qquad CP = 0.5, \qquad MP = 0.1$$

The outcomes of both methods are not the same because the analysis by direct observation shows that only CP and MP are VAPs, whereas analysis by variation transmission shows that PS, CP and MP are VAPs.

The standard deviation from bestfit surface will be 0.628, at PS = 105, CP =0.5 and MP=0.1. Both methods showed that at CP = 0.5 and MP =0.1, the standard deviation will be minimum. Keeping PS = 11, in the equation 4.7 will minimize the average of on-line performance.

So with mean at PS = 11, CP = 0.5 and MP = 0.1 and standard deviation σ_{PS} = 5, σ_{CP} = 0.05, and σ_{MP} = 0.005, final capability study is done to verify the expected improvements.

4.1.2.1 Final Capability Study for Case 1

The on-line performance measured for last 50 % of the generation are shown in Table 4.5, for 20 trials. The control chart for average and standard deviation is also given in Figure 4.3 and Figure 4.4 . The process seems to be capable except two trials falling slightly outside LCL and UCL of average. We now try the next case.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	16.01	15.15	14.51	16.76	15.53	17.41	15.89	18.01	18.13	14.54	16.19	1.34
2	17.77	15.70	22.40	21.03	16.31	20.24	16.61	17.06	17.82	22.11	18.71	2.51
3	20.14	19.44	19.87	14.25	18.39	14.65	13.56	17.72	16.13	20.32	17.45	2.61
4	16.73	14.92	16.97	14.08	21.68	17.16	17.78	16.24	14.62	19.84	17.00	2.35
5	13.88	17.67	16.88	19.60	21.06	21.29	20.74	18.10	16.01	15.34	18.06	2.57
6	18.90	18.58	19.55	19.21	17.93	18.22	18.27	20.48	18.53	18.50	18.82	0.76
7	19.77	21.11	21.28	17.45	20.71	17.27	16.69	20.34	20.80	20.73	19.61	1.77
8	14.82	14.29	19.86	16.84	18.82	20.35	12.97	18.46	16.20	14.76	16.74	2.54
9	22.22	19.46	19.31	20.17	20.19	19.15	20.06	19.88	22.62	19.53	20.26	1.20
10	18.30	14.03	17.05	20.25	13.99	19.57	18.01	14.82	17.63	17.14	17.08	2.18
11	18.99	19.05	19.71	18.85	20.08	19.04	20.21	20.89	17.96	18.62	19.34	0.87
12	17.69	19.59	21.73	17.99	16.93	17.73	17.88	17.24	17.12	17.16	18.11	1.48
13	14.46	12.34	13.06	12.49	12.83	18.31	17.19	14.20	12.18	14.80	14.19	2.10
14	13.84	16.35	16.83	21.38	13.43	20.57	16.56	15.46	17.74	15.45	16.76	2.58
15	15.74	15.81	17.45	19.70	13.67	21.21	20.74	17.77	21.63	15.69	17.94	2.75
16	19.36	22.33	19.49	19.22	18.34	20.24	19.75	17.97	19.41	17.76	19.39	1.31
17	17.00	20.35	20.97	17.30	21.02	18.65	18.88	20.89	19.13	20.34	19.45	1.50
18	15.69	13.97	17.15	19.67	22.29	21.25	13.20	18.57	15.19	13.49	17.05	3.27
19	19.84	22.52	21.37	20.07	22.72	21.61	20.18	20.00	19.47	18.68	20.65	1.34
20	18.72	15.07	18.73	20.11	13.82	14.15	18.04	14.85	13.83	16.87	16.42	2.36

Table 4.5: Final capability study for case 1 function F1

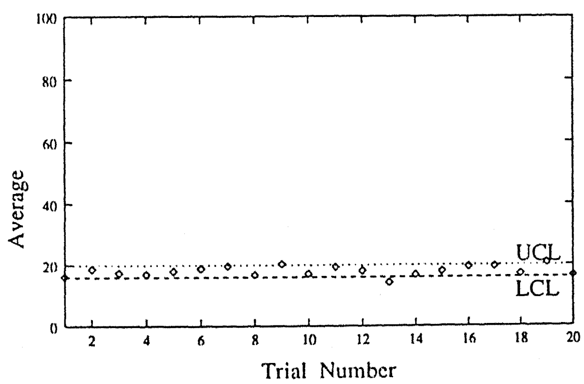

Figure 4.3: Control chart for average of on-line performance after variation reduction for case 1 on function F1

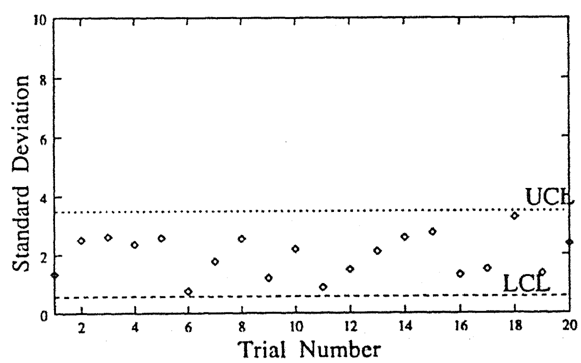

Figure 4.4: Control chart for standard deviation of on-line performance after variation reduction for case 1 on function F1

4.1.3 CASE 2 (Proportionate Selection and Uniform Crossover)

The same way as in case 1, 27 trials of experiment are done with proportionate selection as the selection operator and uniform crossover as the crossover operator. For each trial, the combination of variables are taken from Table 4.6 and ten runs are taken for each trial by changing the initial population. The average and standard deviation are calculated for each trial. The data are shown in Table 4.6.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	112	0.50	0.100	24.7	25.0	25.5	25.1	26.0	24.7	25.0	24.8	24.7	24.5	25.01	0.43
2	112	1.00	0.001	6.9	5.3	21.8	17.7	20.0	16.0	14.4	19.5	19.5	12.4	15.34	5.63
3	11	0.50	0.100	14.4	16.6	17.8	17.8	15.0	12.9	13.3	13.8	14.1	13.2	14.90	1.87
4	112	0.50	0.001	10.0	18.9	15.4	16.9	17.5	18.1	22.9	18.3	15.1	15.6	16.87	3.31
5	11	1.00	0.001	1.4	0.0	1.0	0.8	1.7	0.1	0.0	0.1	0.0	0.0	0.52	0.65
6	11	1.00	0.100	15.4	16.2	17.5	14.3	14.2	15.3	14.6	16.3	13.8	14.4	15.21	1.18
7	112	1.00	0.100	25.3	24.8	25.6	25.5	25.4	25.9	25.7	25.4	25.3	25.2	25.39	0.29
8	11	0.50	0.001	12.9	6.1	7.9	7.4	8.1	7.1	7.1	6.9	7.1	8.0	7.86	1.89
9	61	0.75	0.051	23.8	24.2	22.9	23.0	23.9	23.4	23.3	24.4	23.5	22.7	23.51	0.56
10	61	0.50	0.001	9.1	10.4	1.6	2.3	11.1	2.0	12.0	12.9	7.0	0.9	6.92	4.79
11	61	0.50	0.100	24.1	23.7	23.9	23.6	24.3	24.4	23.7	23.4	24.1	24.3	23.93	0.35
12	61	1.00	0.001	8.6	2.4	2.5	0.3	1.2	3.2	13.2	2.8	1.4	11.3	4.68	4.59
13	61	1.00	0.100	24.4	24.2	24.0	24.0	24.7	24.3	24.4	24.5	24.5	24.1	24.33	0.22
14	11	0.75	0.001	0.0	0.0	0.4	0.2	1.7	0.5	0.4	0.8	0.7	0.8	0.56	0.49
15	11	0.75	0.100	15.7	15.8	13.5	13.9	14.3	13.3	15.3	14.5	14.2	15.7	14.63	0.93
16	112	0.75	0.001	24.1	25.7	22.0	19.0	19.0	18.6	8.6	8.8	12.3	15.0	17.31	5.99
17	112	0.75	0.100	25.0	25.1	25.0	24.7	25.7	25.8	25.6	24.7	25.1	24.7	25.12	0.42
18	11	0.50	0.051	16.4	14.0	18.9	16.9	16.6	9.2	8.2	8.8	8.4	9.0	12.64	4.31
19	11	1.00	0.051	11.3	10.8	18.2	21.2	23.0	22.1	22.0	21.2	22.3	23.3	19.54	4.69
20	112	0.50	0.051	25.9	25.8	25.2	24.5	25.1	24.9	24.7	24.6	24.6	23.9	24.92	0.60
21	112	1.00	0.051	25.3	24.8	25.3	24.0	24.9	24.4	24.8	25.4	25.1	25.2	24.93	0.46
22	61	0.75	0.001	3.1	3.6	7.7	0.8	3.9	0.4	10.2	10.7	15.2	12.0	6.75	5.10
23	61	0.75	0.100	23.8	24.3	23.9	24.0	24.7	24.1	23.8	23.8	24.7	23.5	24.04	0.41
24	11	0.75	0.051	10.0	11.6	23.5	19.7	20.4	19.7	20.0	20.8	21.0	19.9	18.66	4.29
25	112	0.75	0.051	24.8	25.3	24.9	25.1	24.9	24.6	24.4	25.3	24.2	25.1	24.87	0.38
26	61	0.50	0.051	23.9	23.2	21.4	23.0	23.9	23.2	22.6	21.7	23.0	23.3	22.93	0.83
27	61	1.00	0.051	23.1	24.1	22.9	23.5	23.7	23.8	22.7	23.4	24.3	22.3	23.37	0.64

Table 4.6: Data obtained from 27 trials on DeJong F1 function case 2

The effect of each parameter and their interaction with each other is calculated. The quadratic effect of each parameter is also calculated. The significant effects of parameters, their interaction and quadratic effects are shown in Table 4.7 for the average. PS and MP are significant parameters affecting the average. In addition, the interaction PS*MP, CP*MP and the quadratic effect of MP are also found to be significant.

The effect that design parameters have on standard deviation is shown in Table 4.8 . Since MP and PS*MP have significant effects on the variation, this means MP and PS are VAPs.

Design parameter		Effect
MP	mutation prob.	12.86
PS	population size	10.58
MP*MP	mutation prob. (quad)	6.739
CP*MP	crossover prob. * mutation prob.	2.03
PS*MP	population size * mutation prob.	-1.63

Table 4.7: Effects table for average case 2

Design parameter		Effect
MP	mutation prob.	-2.92
PS*MP	population size * mutation prob.	-2.45

Table 4.8: Effects table for standard deviation case 2

Response surface study

Based on 27 trials data in Table 4.6, the best surface through the average is fitted.. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 2.4936 + 0.1159PS + 214.9MP - 1313.87MP^2 \\ &\quad +77.494CP \times MP - .2206PS \times MP \end{aligned} \tag{4.10}$$

The best fit surface through standard deviation is also found using the data of Table 4.6. The resulting equation is :

$$\log(\sigma) = 0.968 - 6.1429MP - 0.1597PS \times MP \tag{4.11}$$

Analysis by direct observation

We see that MP and PS are VAPs as its effect are significant on the standard deviation of the on-line performance. The minimization of the equation 4.11 for the standard deviation gives

$$\sigma = 0.238 \text{ at } PS = 112 \text{ and } MP = 0.1$$

These values of PS and MP can be put into equation 4.10 of average of on-line performance and on-line performance is minimized.

$$\text{Minimum on-line performance} = 25.229 \text{ at } CP = 0.5$$

Based on these analysis we concluded the following:

- Set PS = 112 and MP = 0.1 in order to minimize variation.
- Set CP = 0.5 to reduce the average.

Analysis by Variation transmission

The equation for standard deviation of on-line performance can be found by equation 4.10 of average of on-line performance.

$$\begin{aligned}\sigma^2 &= (.1159 - .2206t_{MP})^2\sigma_{PS}^2 + (77.494t_{MP})^2\sigma_{CP}^2 \\ &+(214.9 - 2\times 1313.87t_{MP} + 77.494t_{CP} - .2206t_{PS})^2\sigma_{MP}^2 + \\ &0.5\times(2\times 1113.87)^2\sigma_{MP}^4 + (77.494)^2\sigma_{CP}\sigma_{MP} + 0.2206^2\sigma_{PS}^2\sigma_{MP}^2\end{aligned} \tag{4.12}$$

Putting $\sigma_{PS} = 5, \sigma_{CP} = 0.05, \sigma_{MP} = 0.005$ in the equation 4.12 and minimizing the equation yields:

$$\sigma = 0.5928 \text{ at } PS = 105,\ CP = 0.54,\ MP = 0.0818.$$

The standard deviation from equation 4.11 will be 0.4 at PS = 105, CP = 0.54 and MP = 0.0818. Comparing the standard deviation found from equation 4.11 from both the method, shows that the the results from the variation transmission method is not bad. By the equation obtained from standard deviation of on-line performance, the transmitted variation can be calculated by the method explained in chapter 2. The transmitted variation are :

$$\sigma_{tPS} = 0.489,\ \sigma_{tCP} = 0.3175,\ \sigma_{tMP} = 0.1066,$$

We see that the transmitted variation by PS and CP is more, so there is need to reduce the variation given to PS and CP as input. Hence PS and CP are VIPs. So with mean at

PS = 105, CP = 0.54 and MP = 0.0818 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$, and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvements.

4.1.3.1 Final Capability Study for Case 2

The data are given in Table 4.9. The control chart for average and standard deviation is also given in Figure 4.5 and Figure 4.6 . The process seems to be capable as all points are within UCL and LCL of average and standard deviation. Also the average and the standard deviation for each trial is reduced and is consistent. We now try the next case.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	25.05	24.81	24.35	25.03	25.06	24.93	24.84	24.90	25.70	25.24	24.99	0.34
2	24.00	24.44	24.91	25.05	24.90	24.69	25.16	25.09	25.24	24.95	24.85	0.35
3	24.64	24.36	25.43	24.92	25.31	23.91	24.00	24.87	24.89	23.63	24.60	0.60
4	24.38	24.06	25.26	25.24	25.03	24.52	24.64	24.65	25.63	25.16	24.86	0.48
5	25.52	24.76	25.04	24.42	24.61	23.73	23.52	24.48	24.58	24.49	24.52	0.57
6	24.42	24.54	25.12	24.69	24.27	25.48	24.07	25.15	24.03	24.81	24.66	0.49
7	23.86	25.02	24.61	25.21	24.79	24.51	24.99	24.52	24.93	25.41	24.79	0.44
8	24.72	24.89	24.17	24.76	24.72	24.76	24.05	24.78	24.80	24.82	24.65	0.29
9	25.01	24.86	25.23	25.12	25.71	24.76	25.25	24.85	25.37	24.33	25.05	0.38
10	24.23	24.99	24.51	24.96	24.69	25.25	25.59	24.88	24.81	24.62	24.85	0.38
11	25.41	25.36	24.99	25.40	25.33	25.22	24.82	25.27	25.00	24.82	25.16	0.23
12	24.86	24.80	25.48	25.46	25.30	24.82	25.63	25.19	25.05	25.55	25.21	0.32
13	25.52	25.20	24.90	24.73	24.33	24.33	25.62	25.06	24.88	24.51	24.91	0.45
14	24.77	24.76	24.70	24.70	24.87	23.95	24.05	24.67	25.02	24.33	24.58	0.35
15	24.94	24.39	24.68	24.78	24.87	25.14	24.79	25.45	25.12	24.40	24.86	0.33
16	24.62	24.93	25.25	24.87	24.96	25.65	23.92	24.53	24.60	25.08	24.84	0.47
17	24.95	24.82	24.39	24.80	25.40	24.04	24.87	24.42	24.75	25.03	24.75	0.38
18	24.60	23.81	24.45	24.54	25.28	24.75	24.19	24.46	25.14	25.24	24.65	0.47
19	25.65	24.87	24.75	24.51	24.96	24.44	24.53	24.89	24.83	24.66	24.81	0.35
20	24.48	25.82	25.20	25.23	24.99	25.33	24.98	25.18	24.58	24.70	25.05	0.40

Table 4.9: Final capability study for case 2 function F1

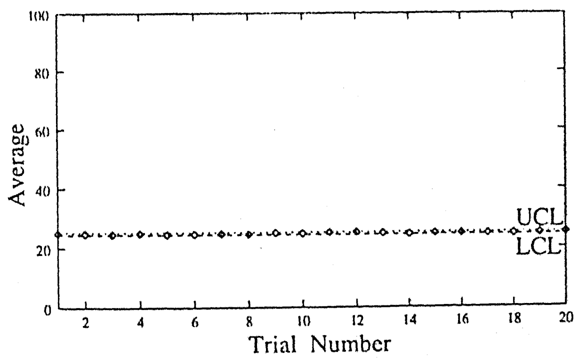

Figure 4.5: Control chart for average of on-line performance after variation reduction case 2 on function F1

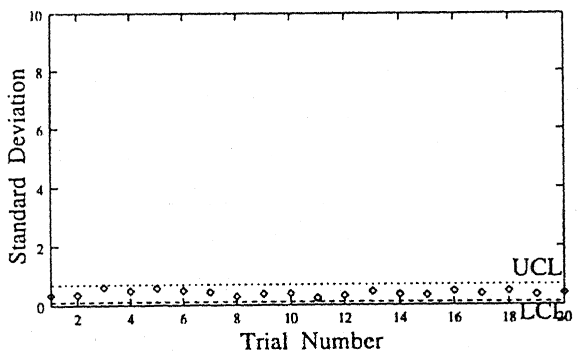

Figure 4.6: Control chart for standard deviation of on-line performance after variation reduction case 2 on function F1

4.1.4 CASE 3 (Tournament Selection and Single Point Crossover)

As in case 2, 27 trials of experiment are done with tournament selection as selection operator and single point crossover as crossover operator. For each trial the variable combination is taken from Table 4.10. For each trial, ten runs are taken by changing the initial population. The average and standard deviation are calculated for each trial. The data are shown in Table 4.10.

Trial no.	candidate input varibles			Run number										On-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	112	0.50	0.100	38.4	37.8	37.5	37.5	38.1	38.8	38.2	38.3	37.7	38.1	38.05	0.41
2	112	1.00	0.001	77.5	77.3	77.6	76.1	77.5	77.2	77.7	77.7	76.1	77.1	77.20	0.61
3	11	0.50	0.100	25.2	22.6	23.2	24.6	23.5	24.0	24.7	22.7	22.8	25.5	23.87	1.08
4	112	0.50	0.001	77.6	77.0	76.9	76.2	76.5	77.1	74.0	77.6	75.9	76.9	76.55	1.07
5	11	1.00	0.001	69.1	48.4	55.2	68.9	53.0	65.9	54.3	50.6	49.1	68.7	58.32	8.75
6	11	1.00	0.100	25.5	24.6	24.5	24.4	22.6	23.6	24.3	25.3	23.2	28.1	24.62	1.53
7	112	1.00	0.100	37.4	36.9	37.5	38.2	38.4	37.2	37.7	38.5	37.3	38.3	37.74	0.57
8	11	0.50	0.001	69.3	57.4	58.3	69.0	56.8	58.8	50.5	51.9	68.9	69.5	61.03	7.48
9	61	0.75	0.051	51.5	51.3	53.0	51.2	51.0	51.2	52.6	54.4	53.0	54.1	52.33	1.27
10	61	0.50	0.001	76.6	75.7	76.8	76.8	75.9	76.2	76.3	76.6	76.8	76.5	76.41	0.39
11	61	0.50	0.100	36.3	37.1	35.7	36.6	35.4	38.3	35.7	38.4	37.4	38.2	36.91	1.16
12	61	1.00	0.001	76.6	75.5	76.9	76.8	76.0	76.8	76.4	76.6	76.5	76.6	76.47	0.43
13	61	1.00	0.100	38.1	37.1	34.6	37.4	36.0	38.5	36.1	37.2	38.0	36.5	36.93	1.17
14	11	0.75	0.001	59.4	58.5	57.6	68.8	49.9	69.0	57.3	49.3	69.2	51.7	59.06	7.71
15	11	0.75	0.100	29.2	30.7	29.2	29.3	33.3	29.8	29.4	28.8	29.1	30.6	29.96	1.35
16	112	0.75	0.001	77.7	77.1	77.7	76.7	77.4	77.6	77.2	77.6	77.2	77.7	77.40	0.32
17	112	0.75	0.100	38.6	38.4	38.2	38.5	38.1	37.8	37.6	39.3	37.7	38.0	38.22	0.51
18	11	0.50	0.051	29.8	30.4	29.4	28.7	28.3	29.3	28.1	28.1	47.4	38.0	31.76	6.24
19	11	1.00	0.051	29.4	29.6	28.4	29.2	28.3	28.6	29.5	31.1	32.6	30.7	29.73	1.37
20	112	0.50	0.051	54.0	53.2	52.1	54.7	54.0	53.7	54.0	53.4	53.3	52.5	53.48	0.79
21	112	1.00	0.051	54.5	54.5	52.8	54.3	53.8	52.3	53.1	54.0	52.3	53.3	53.49	0.86
22	61	0.75	0.001	76.6	76.8	77.0	76.0	75.9	76.8	76.5	76.6	75.1	76.5	76.39	0.57
23	61	0.75	0.100	35.9	37.1	39.6	37.0	36.6	37.7	35.9	38.2	36.6	36.9	37.16	1.11
24	11	0.75	0.051	30.1	29.2	28.1	27.6	30.7	29.5	27.0	30.9	29.8	29.8	29.27	1.31
25	112	0.75	0.051	53.0	52.7	54.1	54.6	53.6	52.1	54.6	54.7	54.5	53.6	53.75	0.91
26	61	0.50	0.051	52.6	53.0	53.1	53.2	50.1	52.8	52.8	54.1	52.3	52.0	52.62	1.05
27	61	1.00	0.051	54.2	54.4	54.0	53.6	52.3	53.3	53.2	50.4	53.4	53.2	53.20	1.15

Table 4.10: Data obtained from 27 trials on DeJong F1 function case 3

The significant effects of parameters, their interaction and quadratic effect is shown in Table 4.11 for the average, and in Table 4.12 for the standard deviation.

Design parameter		Effect
MP	mutation prob.	-37.265
PS	population size	17.58
PS*PS	population size (quad)	7.96
MP*MP	mutation prob. (quad)	-6.83
PS*MP	population size * mutation prob.	-2.86

Table 4.11: Effects table for average case 3

Design parameter		Effect
PS	population size	-3.42
PS*MP	population size * mutation prob.	3.24
MP	mutation prob.	-2.05
PS*PS	population size (quad.)	-1.45

Table 4.12: Effects table for standard deviation case 3

Response surface study

Based on 27 trials data in Table 4.10, we fit the best surface through the average. The resulting equation is :

$$\text{on-line performance} = 52.534 + 0.5977PS - 637.577MP - 0.00315PS^2 + 2982.668MP^2 - 0.6992PS \times MP \tag{4.13}$$

The best fit surface through standard deviation is also found using the data in Table 4.10. The resulting equation is:

$$\log(\sigma) = 2.217 - 0.048PS - 12.75MP + 0.0002PS^2 + 0.1429PS \times MP \tag{4.14}$$

Analysis by direct observation

We see that MP and PS are VAPs as its effect are significant on standard deviation of on-line performance. The minimization of the equation 4.14 for standard deviation gives

$$\sigma = 0.52 \text{ , at } PS = 112 \text{ and } MP = 0.001$$

These value of PS and MP can be put into equation 4.13 of on-line performance then, on-line performance = 79.24

We see that on-line performance doesn't depend on CP. So, CP is not the key input variable. We can keep CP at any point, it will not affect either average or standard deviation. The results from this analysis can be concluded as:

- Set PS = 112 and MP = 0.001 in order to minimize variation.

Analysis by Variation transmission

The equation for standard deviation of on-line performance can be found by equation 4.13.

$$\begin{aligned}\sigma^2 &= (0.5977 - 2 \times 0.00315 t_{PS} - 0.6992 t_{MP})^2 \sigma_{PS}^2 + (-637.577 \\ &\quad +2 \times 2982.668 t_{MP} - 0.6992 t_{PS})^2 \sigma_{MP}^2 + 0.5 \times (2 \times .00315)^2 \sigma_{PS}^4 \\ &\quad +0.5 \times (2 \times 2982.668)^2 \sigma_{MP}^4 + (0.6992)^2 \sigma_{PS}^2 \sigma_{MP}^2 \end{aligned} \tag{4.15}$$

Putting $\sigma_{PS} = 5, \sigma_{CP} = 0.05, \sigma_{MP} = 0.005$, in the equation 4.15 and minimizing it gives

$$\sigma = 0.51 \text{ at } PS = 82,\ MP = 0.1.$$

Putting these values of PS and MP in the equation 4.14 of standard deviation of on-line performance, we get $\sigma = 0.69$. The standard deviation obtained from direct observation method was 0.52.

By the equation 4.15, transmitted variation can be calculated by the method explained in chapter 2. The transmitted variation are :

$$\sigma_{tPS} = 0.1,\ \sigma_{tMP} = 0.5,$$

We see that transmitted variation by MP is more so there is need to reduce the variation given to MP as input. Hence MP is a VIP also.

So with mean at PS = 82, CP = random value between 0.5 to 1, and MP = 0.1 and standard deviation $\sigma_{PS} = 5$ and $\sigma_{MP} = .005$, the final capability study is done to verify the expected improvements.

4.1.4.1 Final Capability Study for Case 3

The data are given in Table 4.13. The control chart for average and standard deviation is also given in Figure 4.7 and Figure 4.8 . The process seems to be capable, as all points are within the UCL and LCL of average and standard deviation. Also the average and standard

deviation of all trials are reduced and are consistent. We now try the next case.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	37.23	36.79	36.76	37.31	38.20	37.72	37.46	37.19	37.10	37.30	37.31	0.42
2	38.04	36.76	37.72	38.58	38.35	37.20	38.11	37.86	38.05	37.83	37.85	0.53
3	38.01	38.05	37.15	39.21	37.72	38.62	39.23	37.75	38.48	38.15	38.24	0.66
4	37.76	38.52	38.16	39.66	36.90	36.44	37.78	37.83	37.89	37.19	37.81	0.89
5	39.62	39.04	39.21	38.94	37.94	37.81	37.74	39.48	38.36	39.02	38.72	0.70
6	36.99	37.30	36.15	36.49	36.41	38.28	38.27	37.07	37.19	37.48	37.16	0.72
7	36.51	37.80	37.07	38.28	36.73	36.95	38.10	36.34	37.34	37.73	37.29	0.67
8	38.37	38.89	39.52	39.59	37.18	38.77	37.20	38.44	36.88	36.96	38.18	1.05
9	38.65	35.76	37.10	36.22	36.83	36.32	36.98	36.07	35.73	37.01	36.67	0.86
10	37.77	38.15	38.90	38.29	38.00	37.68	36.99	37.92	36.96	37.43	37.81	0.59
11	36.57	36.65	36.26	35.90	36.34	36.45	36.41	36.92	36.28	35.93	36.37	0.31
12	36.39	35.65	36.92	36.81	36.77	35.71	36.51	37.00	36.31	36.11	36.42	0.48
13	37.89	37.68	37.83	37.19	36.37	36.67	37.84	37.13	37.74	37.51	37.39	0.53
14	37.73	37.96	39.06	38.63	37.54	37.89	38.42	37.75	38.48	38.34	38.18	0.48
15	36.96	38.54	38.33	37.91	37.75	39.60	39.27	38.49	39.59	37.32	38.38	0.92
16	37.63	36.91	35.52	36.48	36.36	37.19	37.11	37.50	37.24	37.80	36.97	0.69
17	38.51	36.57	37.82	37.76	37.30	36.18	37.47	35.94	36.26	37.31	37.11	0.84
18	38.68	37.79	37.27	39.69	36.91	36.62	39.42	39.61	37.59	39.12	38.27	1.17
19	38.02	35.68	36.67	37.79	36.44	37.10	36.79	37.44	37.22	35.88	36.90	0.77
20	38.03	38.05	38.17	38.36	38.58	37.74	38.73	38.37	37.03	37.75	38.08	0.49

Table 4.13: Final capability study for case 3 function F1

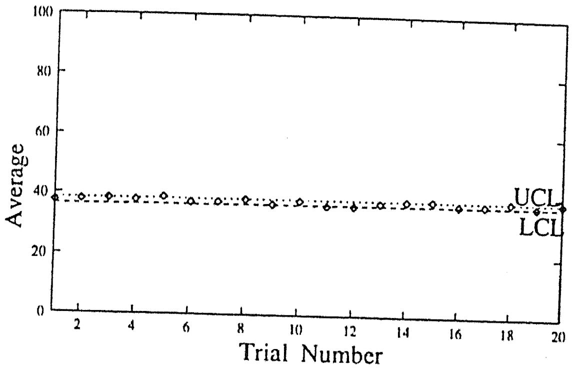

Figure 4.7: Control chart for average of on-line performance after variation reduction case 3 on function F1

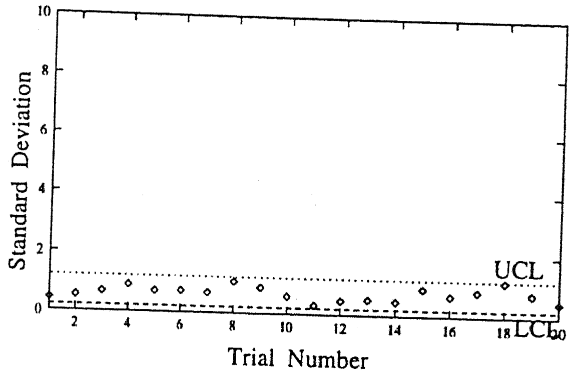

Figure 4.8: Control chart for standard deviation of on-line performance after variation reduction case 3 on function F1

4.1.5 CASE 4 (Tournament Selection and Uniform Crossover)

27 trials of experiment are done with tournament selection as selection operator and uniform crossover as crossover operator. The data are shown in Table 4.14.

The significant effects of parameters, their interaction and quadratic effects are shown in Table 4.15 for average, and in Table 4.16 for standard deviation.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	112	0.50	0.100	34.7	33.9	33.9	33.0	33.8	33.3	34.2	34.4	33.9	34.2	33.93	0.49
2	112	1.00	0.001	77.8	77.7	77.5	77.7	77.7	77.8	77.7	77.7	77.7	77.7	77.70	0.08
3	11	0.50	0.100	22.1	22.1	23.2	23.2	21.9	23.7	23.8	26.5	21.7	23.6	23.17	1.41
4	112	0.50	0.001	77.6	77.7	77.5	77.8	77.7	77.7	77.7	77.5	77.7	74.7	77.36	0.93
5	11	1.00	0.001	68.1	50.2	52.2	56.1	59.1	58.2	55.9	52.2	68.7	59.2	58.00	6.28
6	11	1.00	0.100	25.4	26.1	26.5	27.4	31.2	22.4	27.1	27.9	26.2	27.7	26.78	2.19
7	112	1.00	0.100	31.8	31.1	31.2	30.3	31.5	32.1	31.3	31.2	31.3	31.9	31.37	0.51
8	11	0.50	0.001	69.1	49.2	59.0	68.6	58.5	50.8	57.7	52.6	48.7	54.9	56.89	7.31
9	61	0.75	0.051	53.2	52.8	50.6	52.4	51.1	51.6	50.0	51.9	51.7	52.3	51.76	0.99
10	61	0.50	0.001	76.4	76.6	77.1	76.7	71.8	76.5	76.4	76.4	76.8	76.7	76.13	1.53
11	61	0.50	0.100	33.4	34.4	31.1	33.2	32.1	32.4	33.8	33.1	32.6	34.1	33.02	1.02
12	61	1.00	0.001	76.6	76.8	77.0	76.8	76.0	76.9	76.5	76.5	76.8	76.0	76.57	0.34
13	61	1.00	0.100	31.3	29.4	33.6	29.0	30.3	30.5	30.1	31.0	31.5	31.5	30.83	1.29
14	11	0.75	0.001	68.7	54.9	58.4	59.9	50.3	52.8	54.5	61.4	56.1	48.7	56.56	5.84
15	11	0.75	0.100	25.7	22.9	23.0	26.5	30.7	25.8	27.6	24.5	28.6	27.3	26.26	2.44
16	112	0.75	0.001	77.7	77.7	77.7	77.8	77.7	77.8	77.7	77.8	77.7	77.7	77.73	0.03
17	112	0.75	0.100	31.5	31.9	31.1	32.6	33.9	31.7	32.3	32.9	32.6	31.6	32.21	0.83
18	11	0.50	0.051	28.4	36.1	35.1	37.4	38.6	36.3	36.9	28.6	29.5	36.1	34.31	3.89
19	11	1.00	0.051	25.8	28.0	35.9	35.1	38.0	35.2	29.6	28.4	36.7	46.7	33.95	6.18
20	112	0.50	0.051	55.5	52.8	53.8	53.3	54.6	52.7	53.1	53.0	53.9	53.8	53.65	0.86
21	112	1.00	0.051	54.4	55.4	54.4	52.0	53.4	53.5	55.0	54.1	54.6	54.3	54.11	0.95
22	61	0.75	0.001	76.6	76.8	76.8	76.8	70.2	76.9	76.5	76.5	76.8	76.6	76.05	2.07
23	61	0.75	0.100	31.7	30.7	32.7	30.0	31.9	31.5	31.8	31.6	31.8	31.9	31.57	0.74
24	11	0.75	0.051	28.8	24.6	26.5	26.8	38.7	34.8	35.9	36.0	28.7	36.1	31.69	5.10
25	112	0.75	0.051	55.0	55.1	54.1	53.3	55.1	53.5	53.4	55.1	54.6	54.3	54.34	0.72
26	61	0.50	0.051	53.6	51.8	53.4	52.8	49.6	53.1	52.9	50.5	51.5	52.3	52.15	1.31
27	61	1.00	0.051	49.8	48.7	54.4	50.5	50.7	53.1	50.8	51.7	50.3	52.3	51.23	1.67

Table 4.14: Data obtained from 27 trials on DeJong F1 function case 4

Design parameter		Effect
MP	mutation prob.	-40.428
PS	population size	16.087
PS*MP	population size * mutation prob.	-6.67
PS*PS	population size (quad)	6.589
MP*MP	mutation prob. (quad)	-3.76
PS*CP	population size * crossover prob.	-1.01

Table 4.15: Effects table for average case 4

Design parameter		Effect
PS	population size	-3.91
PS*MP	population size * mutation prob.	2.362
MP	mutation prob.	-1.49
PS*PS	population size (quad.)	-1.33

Table 4.16: Effects table for standard deviation case 4

Response surface study

Based on 27 trials data in Table 4.14, we fit the best surface through the average. The resulting equation is :

$$\text{on-line performance} = 50.29 + 0.5557PS - 338.81MP - 0.0026PS^2 + 116.6MP^2$$
$$-1.32PS \times MP - 0.0119PS \times CP \qquad (4.16)$$

The best fit surface equation through standard deviation is:

$$\log(\sigma) = 2.3788 - 0.0389PS - 13.047MP + 0.00009PS^2 + 0.166PS \times MP \qquad (4.17)$$

Analysis by direct observation

Analysis by direct observation is done in same way as in case 3, i.e. the equation 4.17 is minimized. These values of PS and MP obtained for minimized standard deviation is put into equation 4.16. Then resulting equation in terms of only CP, is minimized for minimum on-line performance. The result from this analysis can be concluded as:

- Set PS = 112 and MP = 0.001 in order to minimize variation.
- Set CP = 1 to reduce the on-line performance.

Analysis by Variation transmission

The equation for standard deviation of on-line performance is obtained from equation 4.16. Putting $\sigma_{PS} = 5, \sigma_{CP} = 0.05, \sigma_{MP} = 0.005$, in the resulting equation and minimizing it gives:

$$\sigma = 2.04 \text{ at } PS = 60, \text{ CP} = 1.0, \text{ and } MP = 0.1.$$

Putting these values of PS and MP in the equation 4.17, we get $\sigma = 1.064$. The value of standard deviation obtained from direct observation analysis is 0.43. This means that result from variation transmission method is not good. This shows that direct observation

result is better for minimum standard deviation. Since we are assuming that the result of variation transmission analysis is not good, hence there is no need to calculate transmitted variation of PS ,CP and MP.

So with mean at PS = 112, CP = 1, and MP = 0.001 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$ and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.1.5.1 Final Capability Study for Case 4

The data are given in Table 4.17. The control chart for average and standard deviation is also given in Figure 4.9 and Figure 4.10. The process seems to be capable except one point is falling slightly outside UCL of standard deviation. We see that average on-line performance is higher compared to case 1, 2 and 3.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	77.26	77.14	77.21	77.26	77.24	77.25	77.17	77.22	77.34	77.26	77.24	0.06
2	77.92	76.33	77.87	77.92	77.92	77.93	77.92	77.92	77.94	77.93	77.76	0.50
3	77.68	77.62	77.62	77.33	77.28	77.66	77.91	77.33	77.59	77.67	77.57	0.20
4	77.60	77.58	77.69	77.62	77.22	77.61	77.61	77.65	77.75	77.68	77.60	0.15
5	77.63	77.59	77.26	77.70	77.27	77.66	77.91	77.33	77.18	77.67	77.52	0.24
6	76.84	76.86	76.61	76.85	76.40	76.82	77.01	76.76	76.74	76.79	76.77	0.16
7	76.79	77.28	77.18	77.22	76.98	77.19	77.14	77.22	77.26	77.21	77.15	0.15
8	77.65	77.62	77.62	77.70	77.28	77.64	77.91	77.33	77.59	77.67	77.60	0.18
9	75.84	75.88	75.50	75.99	75.57	75.92	75.84	75.98	75.78	75.95	75.83	0.17
10	77.92	77.92	77.90	77.92	77.71	77.92	77.91	77.92	77.92	77.93	77.90	0.06
11	75.90	75.74	75.69	75.75	75.93	75.83	75.86	75.88	75.54	75.86	75.80	0.12
12	76.16	76.24	76.31	76.27	76.37	76.30	76.36	76.21	76.20	76.37	76.28	0.08
13	77.62	77.54	77.48	77.54	77.49	77.63	77.53	77.48	77.54	77.56	77.54	0.05
14	77.92	76.33	77.91	77.92	77.92	77.93	77.92	77.92	77.94	77.93	77.76	0.50
15	77.63	77.62	77.26	77.70	77.07	77.66	77.91	77.33	77.18	77.67	77.50	0.27
16	76.82	76.81	76.57	76.80	76.39	76.77	76.97	76.71	76.68	76.74	76.73	0.16
17	76.71	77.25	77.17	77.19	76.95	77.18	77.12	77.15	77.25	77.18	77.12	0.16
18	77.68	77.62	77.60	77.70	77.28	77.64	77.91	77.33	77.18	77.67	77.56	0.22
19	75.76	75.80	75.76	75.94	75.50	75.82	75.73	75.91	75.68	75.86	75.78	0.13
20	77.92	77.92	77.90	77.92	77.92	77.92	77.91	77.88	77.92	77.93	77.91	0.01

Table 4.17: Final capability study for case 4 function F1

4.1.6 Discussion of Result of Function F1

In the Table 4.18 the average and the standard deviation of the final capability study is shown for each case. The value of PS, CP and MP are given at which we have to set it with standard deviation of 5, 0.05 and 0.005 respectively for variation reduction.

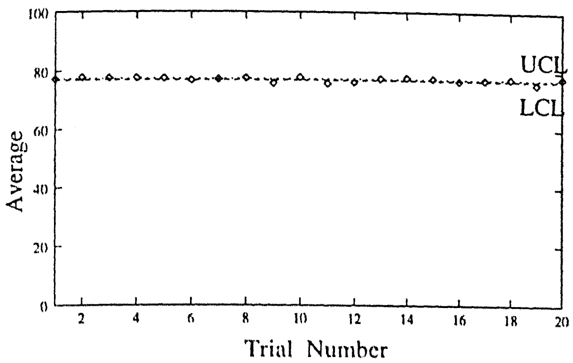

Figure 4.9: Control chart for average of on-line performance after variation reduction case 4 on function F1

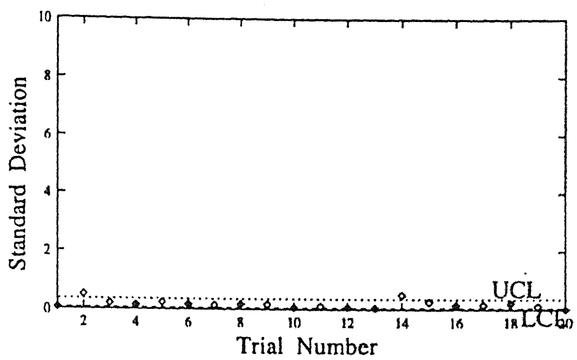

Figure 4.10: Control chart for standard deviation of on-line performance after variation reduction case 4 on function F1

		PS	CP	MP	average	std. deviation
CASE 1	Proportionate selection single point crossover	11	0.5	0.1	17.96	2.086
CASE 2	proportionate selection uniform crossover	105	0.54	0.0818	24.83	0.414
CASE 3	tournament selection single point crossover	82	(0.5 to 1)	0.1	37.55	0.7211
CASE 4	tournament selection uniform crossover	112	1.0	0.001	77.14	0.218

Table 4.18: Result of function F1

We see that if our aim is to minimize only on-line performance then we will choose proportionate selection as selection operator and single point crossover as crossover operator (case 1). But if our aim is to minimize only variation in on-line performance then tournament selection with uniform crossover (case 4) will be chosen. But in this case on-line performance is very high. So we will choose the case in which variation in on-line performance is less and also on-line performance is less. So for function F1 proportionate selection with uniform crossover (case 2) will give better result.

In every case PS, CP and MP which are to be targeted, vary. Since we found that proportionate selection with the uniform crossover is better, we will set PS at 105, CP at 0.54 and MP at 0.081 as mean with standard deviation of 5 for PS, 0.05 for CP and 0.005 for MP. We now see the variation reduction process applied on De Jong Function 2.

4.2 Test Function 2: De Jong Function F2

4.2.1 Initial Capability Study

To check the need for variation reduction a capability study is done. The on-line performance measured for last 50 % of the generation are shown in Table 4.19. The average and standard deviation of each trial are also shown in Table 4.19.

These data are used to construct the average and standard deviation *control charts* shown in Figure 4.11 and Figure 4.12. We see that lot of points are going outside the control limits. So there is need to do variation reduction on this function. For reducing the variation four cases as described in previous chapter is selected and for each case variation reduction technique is applied.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	1254.7	1275.9	1366.7	1331.4	1251.0	1253.5	1175.9	1240.9	1305.7	1124.1	1258.0	70.6
2	220.7	186.4	203.6	160.0	187.4	176.3	188.5	187.8	189.4	195.6	189.6	15.9
3	11.7	50.0	3.7	10.8	2.4	1.7	7.1	5.2	4.8	2.7	10.0	14.5
4	168.7	163.9	151.9	164.4	177.4	180.4	169.8	172.4	158.6	176.7	168.4	8.9
5	25.6	89.9	40.3	90.1	144.8	69.6	104.7	90.2	96.2	135.4	88.7	37.0
6	1493.9	1491.3	1421.7	1489.6	1474.7	1463.1	1410.2	1465.7	1481.3	1518.8	1471.0	33.2
7	273.4	251.9	273.4	264.0	296.0	258.6	265.3	258.8	285.3	291.8	271.8	15.0
8	96.2	100.0	112.3	112.1	108.1	91.0	116.9	116.7	105.0	100.1	105.9	8.9
9	3273.6	3132.2	3249.2	3263.6	3254.2	3279.2	3268.6	3306.6	3247.1	3257.3	3253.2	45.9
10	3201.4	3312.4	3284.6	3311.8	3310.6	3300.1	3306.8	3322.0	3311.5	3274.3	3302.6	14.8
11	1806.0	1812.5	1802.4	1831.3	1759.5	1777.5	1750.3	1838.9	1785.6	1855.5	1808.0	30.2
12	3083.2	3011.5	2989.5	3076.2	3066.9	3037.6	3056.6	3110.9	2980.6	2987.2	3040.0	45.8
13	3509.5	3514.4	3481.6	3496.1	3538.2	3466.0	3514.6	3507.2	3484.5	3467.4	3497.9	23.1
14	224.4	233.4	230.6	237.3	238.1	279.3	236.8	228.6	257.3	248.5	241.4	16.4
15	1381.5	1358.3	1328.2	1392.3	1393.6	1350.5	1372.2	1280.4	1217.8	1328.1	1340.3	55.4
16	2439.0	2406.6	2355.2	2417.5	2344.1	2485.8	2376.5	2499.5	2389.9	2404.7	2411.9	51.1
17	163.9	156.4	202.2	204.2	152.7	173.5	201.0	171.7	168.1	167.8	176.2	19.2
18	77.5	170.3	88.9	36.0	81.2	62.9	36.4	78.7	69.6	40.1	74.2	39.0
19	1563.2	1576.1	1590.4	1543.6	1545.2	1588.7	1536.3	1525.1	1554.0	1525.4	1554.8	24.1
20	1565.9	1542.0	1483.0	1632.5	1626.1	1455.2	1665.9	1493.0	1600.6	1688.0	1575.2	80.5

Table 4.19: Initial capability study for function F2

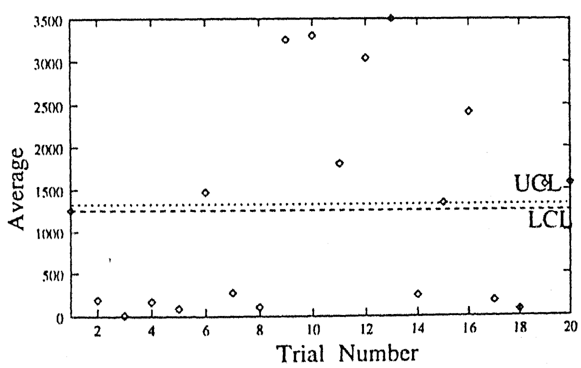

Figure 4.11: Control chart for average on-line performance before variation reduction on function F2

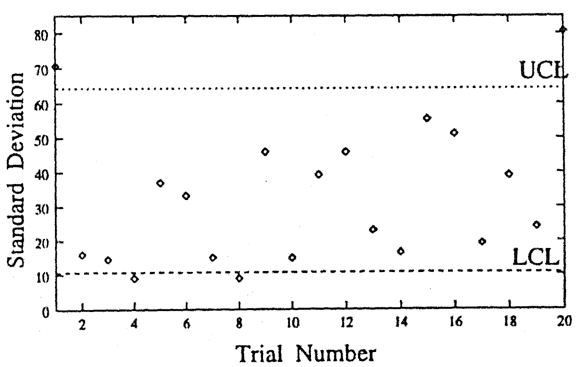

Figure 4.12: Control chart for standard deviation of on-line performance before variation reduction on function F2

4.2.2 CASE 1 (Proportionate Selection and Single Point Crossover)

27 trials of experiment is done with proportionate selection as selection operator and single point crossover as crossover operator. For each trial the combination of variable is taken from Table 4.20 and ten runs are performed by changing the initial population. The average and the standard deviation are calculated for each trial. The data are shown in Table 4.20.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	284.9	298.1	306.0	302.6	285.3	297.9	288.5	308.7	319.0	315.4	300.62	12.00
2	100	1.00	0.001	58.3	79.5	49.1	42.3	52.8	38.4	62.4	14.9	11.6	64.2	47.34	21.43
3	10	0.50	0.100	205.7	135.7	194.8	141.3	146.3	155.5	143.2	137.9	150.3	202.1	161.30	28.03
4	100	0.50	0.001	26.0	28.7	48.7	10.6	121.9	40.2	20.8	77.8	25.5	30.5	43.94	33.03
5	10	1.00	0.001	1.0	11.6	2.6	0.5	6.9	0.1	0.1	1.8	2.4	5.2	3.23	3.69
6	10	1.00	0.100	197.2	239.2	158.3	141.9	147.9	140.1	145.6	301.1	141.0	238.7	185.10	56.66
7	100	1.00	0.100	274.3	306.1	303.6	327.7	328.6	297.7	292.9	317.8	290.1	286.6	302.55	17.88
8	10	0.50	0.001	94.8	2.4	6.4	0.6	223.1	76.9	1.4	0.6	3.6	1.0	41.08	72.92
9	55	0.75	0.051	201.8	196.3	191.5	196.4	199.5	208.6	228.5	198.3	192.1	197.0	201.00	10.84
10	55	0.50	0.001	14.7	22.0	34.1	24.8	81.7	34.6	37.9	21.3	32.8	69.2	37.30	21.56
11	55	0.50	0.100	269.6	254.9	263.2	268.3	252.9	263.4	252.8	258.0	281.6	271.0	263.55	9.29
12	55	1.00	0.001	1.3	0.6	3.8	3.1	80.6	2.3	2.9	1.5	65.2	15.6	17.69	29.64
13	55	1.00	0.100	280.3	249.2	283.3	298.8	248.0	264.6	265.2	279.4	282.3	288.7	273.98	16.70
14	10	0.75	0.001	7.3	83.0	0.5	2.6	1.7	5.9	16.9	0.5	3.9	24.2	14.66	25.23
15	10	0.75	0.100	140.4	157.6	174.3	148.5	134.7	141.7	236.2	168.6	161.8	140.8	160.45	29.71
16	100	0.75	0.001	73.9	29.9	61.8	33.0	88.2	48.3	47.1	48.7	53.6	54.1	53.87	17.49
17	100	0.75	0.100	298.4	293.5	306.9	284.1	278.9	316.6	300.4	301.4	281.6	301.2	296.31	11.89
18	10	0.50	0.051	94.6	197.4	83.3	97.4	112.4	71.3	168.4	218.9	79.8	124.8	124.83	52.11
19	10	1.00	0.051	140.7	117.1	105.0	122.3	75.9	123.2	96.0	133.8	113.5	116.6	114.42	18.65
20	100	0.50	0.051	240.3	209.5	228.9	208.5	240.6	224.9	229.3	194.1	253.9	231.7	226.19	17.80
21	100	1.00	0.051	269.4	250.9	239.3	233.1	243.0	221.3	225.8	240.2	230.5	245.5	239.91	13.79
22	55	0.75	0.001	30.6	54.9	2.4	2.5	49.6	10.0	1.5	11.0	14.3	4.4	18.13	19.97
23	55	0.75	0.100	268.5	264.3	284.7	263.1	280.4	260.0	287.8	251.6	255.8	277.7	269.40	12.55
24	10	0.75	0.051	78.8	75.3	141.3	176.6	133.5	85.2	101.5	97.8	82.3	198.3	117.05	43.51
25	100	0.75	0.051	211.8	226.3	259.0	248.4	235.6	214.6	236.0	211.2	249.2	243.7	233.58	17.02
26	55	0.50	0.051	177.9	160.0	216.0	196.6	183.2	224.5	200.3	228.1	187.6	177.9	195.21	22.21
27	55	1.00	0.051	199.3	192.3	204.0	211.2	208.6	201.1	212.0	203.6	220.7	210.6	206.33	7.99

Table 4.20: Data obtained from 27 trials on DeJong F2 function case 1

The significant effects of parameters, their interaction and the quadratic effects are shown in Table 4.21 for average and in Table 4.22 for standard deviation.

Design parameter		Effect
MP	mutation prob.	215.11
PS	population size	91.35
PS*MP	population size * mutation prob.	51.07
MP*MP	mutation prob. (quad)	45.92
PS*PS	population size (quad)	16.596
CP*MP	crossover prob.* mutation prob.	15.03
PS*CP	population size * crossover prob.	7.25

Table 4.21: Effects table for average case 1

	Design parameter	Effect
CP*MP	crossover prob.* mutation prob.	19.1
PS	population size	18.68
PS*CP	population size*crossover prob.	10.72
PS*PS	population size (quad.)	-10.63
CP	crossover prob.	-9.167
PS*MP	population size* mutation prob.	-7.124
MP	mutation prob.	-5.58
CP*CP	crossover prob. (quad.)	-4.38

Table 4.22: Effects table for standard deviation case 1

Response surface study

Based on 27 trials data in Table 4.20, we fit the best surface through the average. The resulting equation is :

$$\begin{aligned} \text{on-line performance} &= 4 + 1.47PS + 1784.56MP - 0.0081PS^2 - 7332.14MP^2 \\ &\quad -0.2589PS \times CP + 549.91CP \times MP \\ &\quad +12.58PS \times MP \end{aligned} \tag{4.18}$$

The best fit surface through standard deviation is:

$$\begin{aligned} \log(\sigma) &= 16.37 - 0.044PS - 29.32CP - 58.4MP + 0.000197PS^2 + 15.051CP^2 \\ &\quad +0.029PS \times CP + 82.4CP \times MP - 0.1PS \times MP \end{aligned} \tag{4.19}$$

Analysis by direct observation

The result from this analysis can be concluded as:

- Set PS = 90, CP = 0.64 and MP = 0.0991 in order to minimize variation.

Analysis by Variation transmission

Minimization of the equation for standard deviation obtained from equation 4.18 gives

$$\sigma = 10.43 \text{ at } PS = 82,\ CP = 0.5,\ MP = 0.1$$

The standard deviation from equation 4.19 will be 5.98, at PS = 82, CP =0.5 and MP=0.1, while standard deviation from direct observation method was 4.773. This means that result from analysis of variation transmission method is not bad. The transmitted variation of each parameter was found to be:

$$\sigma_{tPS} = 6.318, \sigma_{tCP} = 1.694, \sigma_{tMP} = 8.13$$

This shows that PS and MP are VIPs. So there is need to reduce the variation of PS and MP. So with mean at PS = 82, CP = 0.5 and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$, and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.2.2.1 Final Capability Study for Case 1

The data are shown in Table 4.23. The control chart for average and standard deviation is also given in Figure 4.13 and Figure 4.14. The process is capable as all points are within control limits except one point which is slightly out of UCL limit in control chart for standard deviation. We now try the next case.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	299.1	295.2	301.1	290.9	294.2	309.9	286.7	287.9	324.5	305.9	299.6	11.5
2	305.9	304.9	278.9	276.4	289.5	279.1	279.0	277.3	291.0	297.4	287.9	11.5
3	296.2	296.9	315.1	272.3	256.5	272.4	283.4	266.1	263.1	299.8	282.2	19.1
4	297.5	285.2	281.4	280.8	301.3	295.8	280.6	267.7	284.6	296.9	287.2	10.4
5	275.9	292.7	282.3	286.5	274.6	278.7	261.7	282.6	271.1	284.0	279.0	8.7
6	302.7	291.5	273.2	323.0	307.8	321.4	292.3	322.6	290.5	306.8	303.2	16.5
7	303.3	284.7	294.1	291.0	311.8	294.0	306.9	315.2	290.5	289.9	298.1	10.4
8	264.3	280.5	293.8	271.0	295.9	279.1	283.8	285.9	268.9	288.4	281.2	10.5
9	304.3	287.9	297.4	314.9	302.2	315.4	299.2	308.7	286.2	301.2	301.8	9.9
10	285.1	311.8	286.5	325.5	330.3	324.7	301.4	283.2	339.7	278.7	306.7	22.6
11	301.1	284.4	317.1	297.9	305.0	338.7	308.2	306.1	298.9	324.3	308.2	15.3
12	312.4	299.3	319.9	323.4	314.2	328.5	308.0	294.2	326.6	293.2	312.0	13.0
13	298.2	320.1	270.9	300.3	288.1	306.0	306.6	312.7	294.5	306.3	300.4	13.7
14	304.2	280.6	318.9	268.7	283.5	287.8	297.1	297.7	323.3	252.7	291.4	21.6
15	287.1	265.6	283.3	280.8	268.1	328.3	351.8	293.7	259.8	289.4	290.8	28.7
16	286.8	311.4	307.7	294.6	311.3	316.5	310.4	310.2	286.2	298.5	303.3	11.0
17	298.0	278.1	292.7	301.9	291.0	303.3	294.6	269.6	327.7	287.0	294.4	15.7
18	288.8	290.5	267.6	291.6	284.4	291.6	268.4	296.7	278.8	295.9	285.4	10.6
19	313.1	285.0	315.1	309.2	321.1	269.9	300.1	308.2	280.0	319.5	302.1	17.9
20	279.4	292.6	293.3	286.1	299.9	302.3	291.3	276.1	312.7	278.6	291.2	11.6

Table 4.23: Final capability study for case 1 function F2

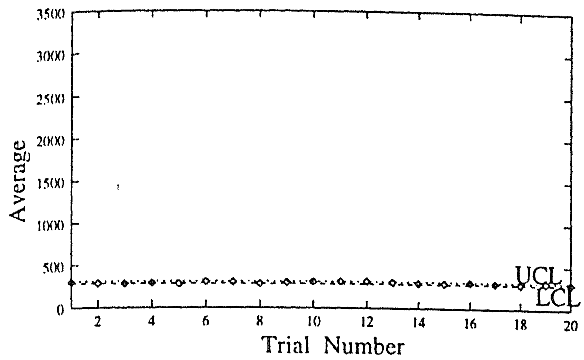

Figure 4.13: Control chart for average of on-line performance after variation reduction for case 1 on function F2

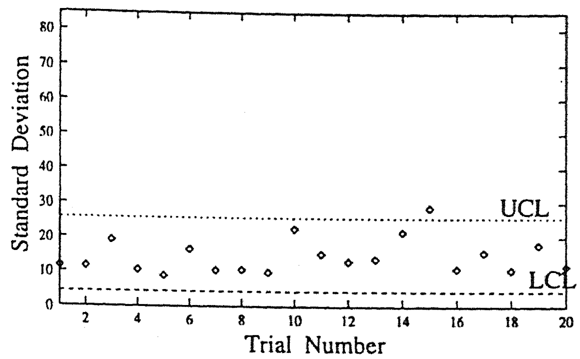

Figure 4.14: Control chart for standard deviation of on-line performance after variation reduction for case 1 on function F2

4.2.3 CASE 2 (Proportionate Selection and Uniform Crossover)

27 trial experiment data are shown in Table 4.24.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	322.0	313.7	316.4	337.4	337.4	338.3	322.2	309.2	328.9	324.2	324.98	10.39
2	100	1.00	0.001	62.4	70.8	140.8	17.2	73.4	40.7	137.9	49.6	17.4	139.6	74.98	48.44
3	10	0.50	0.100	220.7	167.5	239.0	158.6	148.7	149.5	220.2	152.6	213.2	216.7	188.67	36.10
4	100	0.50	0.001	47.5	35.7	48.2	112.6	138.0	103.9	141.0	5.9	94.1	69.2	79.62	45.41
5	10	1.00	0.001	24.3	37.0	2.3	0.1	22.4	494.5	11.1	0.4	0.1	0.6	59.28	153.46
6	10	1.00	0.100	160.6	157.3	168.0	159.1	156.3	186.2	160.6	243.8	156.1	164.2	171.22	27.01
7	100	1.00	0.100	343.6	338.9	354.1	292.1	315.3	330.3	343.6	340.2	334.7	341.5	333.44	17.72
8	10	0.50	0.001	625.5	1.0	2.3	0.3	0.4	15.4	34.1	323.8	14.2	84.9	110.19	206.43
9	55	0.75	0.051	238.2	230.7	224.7	253.5	216.6	230.0	233.3	234.1	234.5	234.2	232.98	9.47
10	55	0.50	0.001	95.2	41.4	9.4	1.1	8.0	100.6	34.3	52.8	7.8	59.3	40.98	36.20
11	55	0.50	0.100	311.7	273.7	288.5	284.3	265.3	295.7	288.6	263.1	283.5	291.0	284.55	14.48
12	55	1.00	0.001	27.8	44.7	25.1	56.5	34.5	94.2	41.9	44.2	28.0	47.5	44.44	20.14
13	55	1.00	0.100	289.4	308.0	296.7	293.0	322.5	301.8	324.0	290.6	310.0	296.2	303.23	12.56
14	10	0.75	0.001	571.5	0.1	3.5	4.6	8.9	7.0	1.3	0.3	0.2	2.3	59.97	179.75
15	10	0.75	0.100	154.2	178.2	155.2	220.6	160.2	162.1	205.4	157.8	174.6	239.4	181.36	31.34
16	100	0.75	0.001	43.2	46.9	47.8	28.4	27.5	37.6	137.6	35.0	91.7	29.9	52.56	35.23
17	100	0.75	0.100	319.1	301.4	348.5	330.2	345.5	333.7	336.7	336.9	333.7	318.9	330.47	13.98
18	10	0.50	0.051	92.6	119.2	100.1	254.9	170.4	97.9	92.5	158.8	98.7	129.6	131.47	51.49
19	10	1.00	0.051	83.0	106.1	145.1	76.5	94.3	93.8	176.1	88.4	126.9	101.1	109.14	31.27
20	100	0.50	0.051	292.0	291.9	258.7	247.9	275.6	302.0	250.8	294.8	238.8	289.5	274.20	23.08
21	100	1.00	0.051	275.1	289.9	299.2	268.3	256.6	268.3	268.8	268.8	280.8	305.0	278.08	15.47
22	55	0.75	0.001	103.3	42.2	27.4	20.4	15.0	29.1	0.7	3.4	2.6	5.2	25.55	30.71
23	55	0.75	0.100	298.9	291.2	293.0	302.3	296.0	293.4	299.2	282.8	317.9	287.6	296.24	9.56
24	10	0.75	0.051	93.9	154.1	153.8	100.7	268.0	98.7	92.5	102.7	129.0	188.7	138.21	56.03
25	100	0.75	0.051	280.6	261.6	279.6	258.1	262.4	257.6	281.2	272.8	269.6	283.3	270.67	10.18
26	55	0.50	0.051	215.9	245.7	216.7	238.1	209.0	224.6	227.8	220.0	217.3	238.2	225.33	11.91
27	55	1.00	0.051	215.1	226.7	214.9	259.8	247.1	241.4	270.2	219.7	260.1	252.4	240.74	20.41

Table 4.24: Data obtained from 27 trials on DeJong F2 function case 2

The significant effects of parameters, their interaction and the quadratic effects are shown in Table 4.25 for average and in Table 4.26 for standard deviation.

Design parameter		Effect
MP	mutation prob.	207.4
PS	population size	96.6
PS*MP	population size * mutation prob.	78.32
MP*MP	mutation prob. (quad)	46.66
PS*CP	population size * crossover prob.	16.39
PS*PS	population size (quad)	12.19
CP*MP	crossover prob. * mutation prob.	10.29

Table 4.25: Effects table for average case 2

Table 4.26: Effects table for standard deviation case 2

Design parameter		Effect
MP	mutation prob.	-64.73
PS	population size	-61.44
PS*MP	population size * mutation prob.	59.7
PS*PS	population size (quad)	-36.77
MP*MP	mutation prob. (quad)	-26.127
PS*CP	population size * crossover prob.	14.17
CP*MP	crossover prob. * mutation prob.	10.38
CP	crossover prob.	-9.88

Response surface study

Based on 27 trials data in Table 4.24, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 51.015 + 0.8585PS + 1639.9MP - 0.006PS^2 - 8034.29MP^2 \\ &\quad -0.082PS \times CP + 281.23CP \times MP \\ &\quad +18.725PS \times MP \end{aligned} \tag{4.20}$$

The best fit surface through the standard deviation is:

$$\begin{aligned}\log(\sigma) &= 6.71 - 0.066PS - 1.619CP - 30.08MP + 0.00039PS^2 + 22.237MP^2 \\ &\quad +0.01PS \times CP + 15.546CP \times MP + 0.065PS \times MP \end{aligned} \tag{4.21}$$

Analysis by direct observation

- Set PS = 55 , CP = 0.5 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of the equation for standard deviation obtained from equation 4.20 gives

$$\sigma = 9.76 \text{ at } PS = 10,\ CP = 0.5,\ MP = 0.02$$

The standard deviation from equation 4.21 will be 133.38 at PS = 10, CP = 0.5 and MP = 0.02, while value of standard deviation obtained from direct observation method is 7.917. This means the result from Variation method method is not good.

So with mean at PS = 55, CP = 0.5 and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$, and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.2.3.1 Final Capability Study for Case 2

The data are given in Table 4.27. The control chart for average and standard deviation is also given in Figure 4.15 and Figure 4.16. The process is capable as all points fall within the control limits. Also the standard deviation is less compared to initial capability study. We now try the next case.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	300.0	274.8	279.5	283.1	291.2	312.1	294.8	297.8	299.8	282.1	291.5	11.5
2	276.9	293.3	310.5	282.7	285.2	272.9	280.6	305.3	303.0	269.7	288.0	14.3
3	291.8	314.9	314.5	278.9	346.2	301.1	275.7	288.8	284.2	297.9	299.4	21.2
4	304.2	281.1	304.6	319.8	319.6	296.9	302.3	312.9	280.8	282.2	300.4	15.0
5	302.2	287.5	327.0	293.4	302.5	274.8	284.6	283.1	275.4	265.2	289.6	17.7
6	300.9	310.4	310.0	341.8	316.8	307.2	318.6	308.1	292.7	300.4	311.6	13.6
7	300.5	295.2	295.6	298.6	346.4	340.0	300.6	273.7	287.7	277.4	301.6	23.9
8	251.5	268.7	271.4	265.5	304.5	260.9	282.8	295.2	250.5	274.1	272.5	17.6
9	314.2	311.5	299.6	317.2	317.6	325.7	309.2	315.0	318.8	307.2	313.6	7.2
10	333.8	287.4	298.5	278.3	302.4	324.3	290.0	285.2	281.1	296.2	294.7	20.5
11	292.4	303.0	280.2	322.9	311.1	314.4	319.9	357.8	333.2	320.0	315.5	21.5
12	314.5	282.1	287.1	300.9	302.8	293.0	300.0	286.4	288.7	303.1	295.9	10.0
13	298.4	287.6	285.5	297.1	297.7	295.1	326.7	288.7	311.5	312.0	300.0	13.0
14	291.0	290.7	300.4	287.2	285.4	281.5	298.3	318.7	293.2	290.3	290.7	14.9
15	291.5	288.2	278.0	272.2	276.1	274.7	269.2	266.3	289.1	298.7	280.4	10.8
16	306.9	299.0	307.4	296.6	293.0	297.8	308.2	276.4	299.9	305.0	299.0	9.5
17	306.7	307.2	281.2	300.2	308.9	311.3	273.8	292.7	298.9	286.9	296.8	12.8
18	254.4	256.4	273.4	259.2	263.3	279.1	280.9	269.7	276.1	270.0	268.2	9.5
19	330.9	320.5	313.6	302.8	311.8	309.5	332.7	294.9	313.5	321.2	315.1	11.7
20	303.7	297.1	300.4	292.9	294.4	316.2	299.8	263.9	292.7	270.5	293.2	15.4

Table 4.27: Final capability study for case 2 function F2

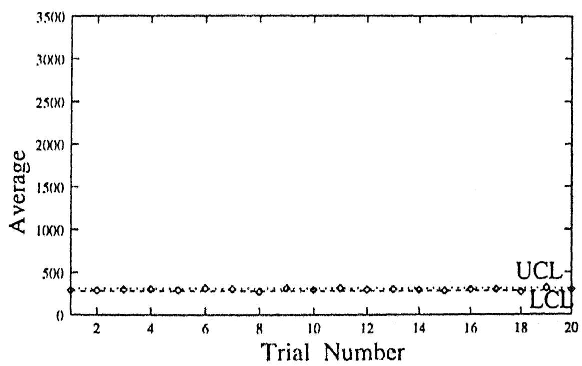

Figure 4.15: Control chart for average of on-line performance after variation reduction case 2 on function F2

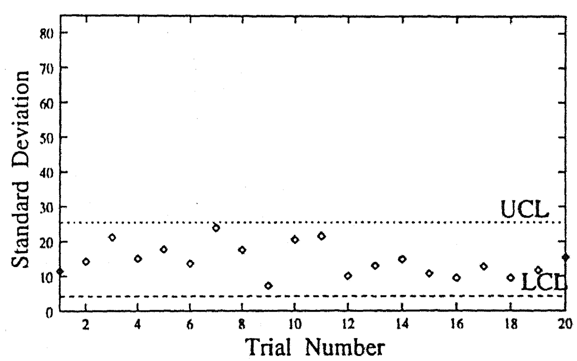

Figure 4.16: Control chart for standard deviation of on-line performance after variation reduction case 2 on function F2

4.2.4 CASE 3 (Tournament Selection and Single Point Crossover)

27 trials experiment data are shown in table 4.28.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	1339.2	1425.7	1444.6	1447.4	1424.4	1415.1	1400.7	1438.0	1371.6	1439.5	1414.62	35.12
2	100	1.00	0.001	3770.7	3831.1	3829.9	3832.6	3691.3	3844.4	3851.8	3851.9	3823.8	3841.0	3816.84	49.83
3	10	0.50	0.100	676.3	591.1	686.7	674.5	653.0	574.3	608.4	659.3	544.8	616.6	628.50	48.56
4	100	0.50	0.001	3803.0	3818.4	3685.7	3642.2	3842.4	3852.9	3804.7	3672.6	3792.5	3752.9	3766.74	74.81
5	10	1.00	0.001	2900.4	3417.6	3361.2	3348.7	1220.4	3186.6	3347.6	2280.6	2508.5	3463.7	2903.53	718.02
6	10	1.00	0.100	625.0	673.0	664.2	621.5	641.2	606.2	588.5	615.5	591.0	643.5	626.98	28.59
7	100	1.00	0.100	1409.2	1344.1	1458.7	1482.1	1406.5	1384.3	1427.8	1431.2	1395.4	1442.1	1418.14	39.35
8	10	0.50	0.001	3467.8	3014.2	2878.5	2783.9	2816.7	2874.1	1369.5	1381.7	1192.4	3363.4	2514.23	859.41
9	55	0.75	0.051	2487.1	2447.0	2417.7	2472.0	2417.2	2399.0	2518.6	2408.2	2325.8	2454.6	2434.73	53.91
10	55	0.50	0.001	3782.1	3669.3	3575.0	3223.9	3546.7	3025.0	3795.5	3661.8	3084.0	3681.6	3504.49	286.21
11	55	0.50	0.100	1322.9	1264.7	1218.2	1296.2	1309.7	1364.3	1272.6	1244.5	1272.0	1205.2	1277.02	48.34
12	55	1.00	0.001	3800.1	3722.3	3744.6	3484.5	3019.9	3742.8	3659.5	3268.2	3658.7	3701.4	3580.20	251.57
13	55	1.00	0.100	1359.1	1213.5	1257.1	1204.8	1251.9	1172.7	1273.9	1251.8	1304.3	1262.0	1255.11	52.42
14	10	0.75	0.001	2208.7	2389.1	3315.9	3132.5	2262.6	3472.5	1992.1	3350.0	3307.1	3303.9	2873.44	582.02
15	10	0.75	0.100	638.1	637.7	673.5	681.1	646.9	583.7	597.4	689.1	589.3	649.6	638.65	37.88
16	100	0.75	0.001	3836.3	3685.0	3828.5	3846.1	3841.0	3843.4	3714.2	3852.4	3836.0	3837.9	3812.08	60.02
17	100	0.75	0.100	1526.6	1442.9	1434.3	1463.1	1454.8	1436.6	1387.9	1465.6	1345.8	1451.0	1440.87	47.91
18	10	0.50	0.051	1159.9	999.1	1208.6	1229.8	1300.3	1238.3	1127.0	1200.1	1195.7	1200.9	1185.97	80.15
19	10	1.00	0.051	1233.1	1319.7	1130.2	1413.6	1314.4	1200.8	1152.3	1361.9	1180.8	1257.3	1256.42	93.89
20	100	0.50	0.051	2674.3	2662.8	2585.8	2696.5	2610.6	2651.7	2626.0	2657.2	2595.6	2630.8	2639.14	35.55
21	100	1.00	0.051	2632.0	2523.2	2612.5	2693.5	2650.9	2554.9	2579.1	2662.3	2606.5	2662.2	2617.72	53.26
22	55	0.75	0.001	3807.2	3775.0	3774.6	3784.9	3078.6	3735.2	3789.4	3788.2	3774.7	3765.1	3707.28	221.68
23	55	0.75	0.100	1354.3	1260.3	1209.0	1315.2	1153.1	1191.1	1257.8	1289.1	1135.4	1256.0	1242.12	69.85
24	10	0.75	0.051	1191.2	1274.7	1236.7	1187.8	1224.3	1131.1	1132.6	1263.7	1376.8	1238.8	1225.78	72.44
25	100	0.75	0.051	2636.6	2433.2	2609.5	2579.3	2402.1	2603.4	2607.5	2582.2	2523.5	2651.6	2562.89	84.32
26	55	0.50	0.051	2479.5	2413.4	2428.0	2533.2	2404.0	2472.8	2384.5	2441.2	2351.2	2443.4	2435.11	51.87
27	55	1.00	0.051	2429.8	2557.2	2401.0	2463.5	2464.0	2490.3	2562.5	2562.8	2402.5	2406.6	2474.01	66.72

Table 4.28: Data obtained from 27 trials on DeJong F2 function case 3

The significant effects of parameters, their interaction and the quadratic effects are shown in Table 4.29 for average and in Table 4.30 for standard deviation.

Design parameter		Effect
MP	mutation prob.	-2281.87
PS	population size	1070.61
PS*PS	population size (quad)	359.87
MP*MP	mutation prob. (quad)	-153.184
PS*MP	population size * mutation prob.	-120.83
CP*MP	crossover prob. * mutation prob.	-89.17
PS*CP	population size * crossover prob.	-71.006
CP	crossover prob.	64.8
CP*CP	crossover prob.(quad.)	31.16

Table 4.29: Effects table for average case 3

Design parameter		Effect
PS*MP	population size * mutation prob.	330.35
MP	mutation prob.	-299.5
PS	population size	-266.75
MP*MP	mutation prob. (quad)	-129.29
PS*PS	population size (quad.)	-44.22
CP*MP	crossover prob. * mutation prob.	31.55
PS*CP	population size * crossover prob.	24.09
CP*CP	crossover prob.(quad.)	-22.98
CP	crossover prob.	-18.48

Table 4.30: Effects table for standard deviation case 3

Response surface study

Based on 27 trials data in Table 4.28, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 1084.62 + 35.83PS + 2886.36CP - 19469.5MP - 0.178PS^2 \\ &\quad -1516.32CP^2 + 22060.7MP^2 - 3.65PS \times CP \\ &\quad -5342.28CP \times MP - 30.91PS \times MP \qquad (4.22)\end{aligned}$$

The best fit surface through standard deviation is:

$$\begin{aligned}\log(\sigma) &= 3.968 - 0.017PS + 7.19CP - 36.78MP - 0.000071PS^2 - 4.097CP^2 \\ &\quad +102.26MP^2 + 0.0022PS \times CP - 5.29CP \times MP + 0.264PS \times MP (4.23)\end{aligned}$$

Analysis by direct observation

The results from this analysis can be concluded as:

- Set PS = 10 , CP = 1 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of the equation for standard deviation of on-line performance obtained by equation 4.22 gives:

$$\sigma = 105.13 \text{ at } PS = 82, \text{CP} = 0.57 \text{ and } MP = 0.1.$$

Putting the value PS = 82, CP = 0.57 and MP = 0.1 in the equation 4.23 we get $\sigma = 51.2$. The value of standard deviation obtained by direct observation method is 29.34. This means that the result from variation transmission method is not good.

So with mean at PS = 10, CP = 1, and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$ and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.2.4.1 Final Capability Study for Case 3

The data are given in Table 4.31. The control chart for average and standard deviation is also given in Figure 4.17 and Figure 4.18. The process in this case is not capable since some points in control chart are marginally out of the control limits. This means that tournament selection with single point cross over for this function does not gives capable process. We now try for next case.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	733.0	762.8	627.0	744.1	711.6	678.5	725.0	698.0	693.1	772.6	714.7	43.2
2	528.7	470.5	571.0	549.6	532.1	546.5	571.0	491.8	525.8	516.2	530.3	32.0
3	556.5	578.0	502.3	531.5	539.0	524.6	465.2	596.2	.499.3	512.9	530.5	39.0
4	378.2	391.5	275.7 ·	348.1	372.0	323.5	377.2	345.7	390.8	395.4	359.8	37.6
5	508.2	567.8	591.6	492.1	580.2	601.7	503.8	571.8	533.3	596.9	554.7	41.6
6	479.2	590.5	596.2	576.8	532.6	589.1	541.0	592.5	518.9	474.8	549.2	46.9
7	725.0	692.0	711.0	734.9	712.3	667.6	658.7	709.8	781.6	733.4	712.6	35.3
8	524.5	509.9	521.1	547.2	508.9	513.5	499.6	591.6	483.9	539.8	530.0	32.7
9	843.2	857.8	824.9	889.4	865.8	793.8	855.7	861.5	834.9	835.5	846.3	26.2
10	787.6	827.8	823.7	815.7	873.3	807.8	872.0	839.9	793.0	822.9	826.4	29.0
11	613.2	620.9	647.7	606.1	688.9	627.3	623.0	646.6	613.9	587.1	628.4	28.0
12	651.0	535.2	560.1	558.1	529.7	842.7	928.3	601.7	645.3	886.6	673.9	153.2
13	693.5	471.7	469.2	829.2	777.3	744.7	936.1	754.4	881.4	811.7	736.9	156.5
14	545.4	548.2	566.6	599.3	531.9	508.9	546.8	599.9	558.3	599.4	560.5	31.0
15	711.0	681.8	703.8	753.8	680.0	659.0	640.6	748.2	800.3	671.5	705.0	49.3
16	740.7	791.3	812.7	805.5	789.6	774.3	793.4	757.4	742.8	778.8	778.6	24.9
17	528.8	558.6	509.0	552.4	525.4	568.9	534.2	534.9	579.0	538.9	543.0	21.4
18	555.5	566.3	571.2	524.7	611.6	533.9	560.6	521.7	521.6	519.5	548.7	29.9
19	766.2	828.9	572.4	952.7	492.1	868.9	492.0	536.8	889.6	853.8	725.3	181.4
20	347.8	382.0	342.0	306.6	385.1	294.5	397.2	342.3	369.4	1173.3	434.0	261.9

Table 4.31: Final capability study for case 3 function F2

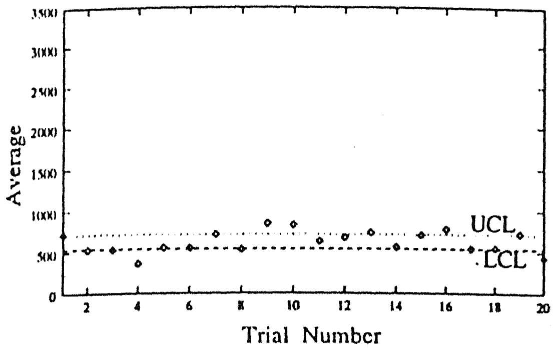

Figure 4.17: Control chart for average of on-line performance after variation reduction case 3 on function F2

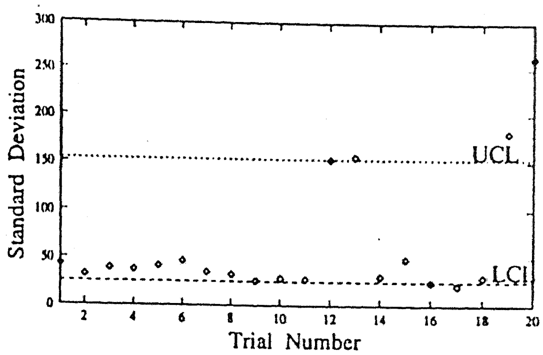

Figure 4.18: Control chart for standard deviation of on-line performance after variation reduction case 3 on function F2

4.2.5 CASE 4 (Tournament Selection and Uniform Crossover)

27 trials experiment data are shown in Table 4.32.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	1304.2	1424.4	1395.9	1323.3	1365.2	1385.8	1369.7	1453.3	1420.6	1367.7	1381.02	45.60
2	100	1.00	0.001	3844.4	3856.2	3846.4	3844.1	3853.9	3844.0	3855.0	3849.8	3841.3	3844.0	3847.93	5.41
3	10	0.50	0.100	602.5	597.7	592.7	570.5	620.3	528.4	600.6	599.6	611.7	618.9	594.29	27.15
4	100	0.50	0.001	3842.1	3841.8	3840.3	3838.0	3854.8	3835.1	3788.4	3847.2	3849.3	3840.9	3837.80	18.28
5	10	1.00	0.001	3332.3	2662.5	3193.4	3349.2	1752.6	3440.7	1979.2	2693.6	3421.3	2627.8	2845.26	609.79
6	10	1.00	0.100	538.7	555.7	626.7	620.3	595.6	532.9	497.2	591.6	589.8	574.6	572.30	40.94
7	100	1.00	0.100	1372.6	1323.6	1328.4	1346.5	1417.0	1461.2	1387.6	1372.2	1438.7	1401.7	1384.96	45.67
8	10	0.50	0.001	3432.6	3183.3	3098.1	1808.9	2255.0	2410.8	3470.7	1550.5	3297.5	3325.7	2783.90	714.47
9	55	0.75	0.051	2020.7	2390.0	2418.9	2397.1	2375.9	2395.1	2348.8	2422.4	2529.3	2374.4	2428.15	85.89
10	55	0.50	0.001	3803.4	3654.4	3735.0	3673.9	3684.9	3736.4	3752.5	3714.4	3154.7	3721.0	3663.04	183.58
11	55	0.50	0.100	1380.5	1209.4	1217.2	1065.9	1363.6	1336.5	1329.0	1284.3	1133.3	1400.9	1272.07	112.04
12	55	1.00	0.001	3794.3	3732.7	3664.5	3594.8	3743.1	3686.3	3747.0	3730.7	3741.0	3733.4	3716.78	55.23
13	55	1.00	0.100	1177.9	1162.3	1244.9	1197.2	1228.7	1185.0	1144.7	1253.2	1200.6	1186.4	1198.09	35.00
14	10	0.75	0.001	3320.4	3175.7	3482.3	3178.9	2373.5	3153.1	3058.1	3185.0	3370.6	2991.1	3128.87	302.41
15	10	0.75	0.100	575.0	508.2	653.6	551.1	594.8	594.7	556.2	567.1	600.5	613.0	587.42	30.80
16	100	0.75	0.001	3852.0	3848.5	3830.3	3841.6	3851.1	3845.8	3853.7	3854.6	3846.3	3838.7	3847.16	5.82
17	100	0.75	0.100	1309.1	1352.1	1434.6	1336.3	1339.7	1308.2	1362.5	1337.8	1268.2	1418.9	1358.74	46.10
18	10	0.50	0.051	1180.6	1215.1	1182.7	1210.9	1339.4	1285.5	1175.1	1216.3	1164.5	1208.1	1210.02	54.09
19	10	1.00	0.051	1160.2	1227.8	1357.1	1167.6	1448.8	1119.3	1261.3	1251.5	1245.5	1220.9	1246.00	96.75
20	100	0.50	0.051	2590.6	2635.9	2664.0	2620.3	2670.6	2637.5	2632.4	2632.3	2648.4	2642.9	2637.51	22.35
21	100	1.00	0.051	2630.3	2662.4	2654.0	2631.0	2720.8	2625.5	2626.4	2578.6	2629.7	2670.4	2642.92	37.31
22	55	0.75	0.001	3614.3	3732.8	3737.7	3748.3	3160.1	3735.2	3736.4	3748.6	3748.9	3735.7	3669.80	183.53
23	55	0.75	0.100	1267.2	1130.8	1179.0	1194.3	1064.3	1171.6	1184.2	1159.7	1170.1	1186.8	1170.81	51.01
24	10	0.75	0.051	1443.7	1108.8	1212.3	1333.1	1281.1	1234.8	1191.9	1279.6	1090.1	1384.9	1256.04	112.85
25	100	0.75	0.051	2603.1	2585.7	2565.6	2652.4	2640.7	2654.9	2638.7	2661.0	2548.8	2633.1	2618.39	39.96
26	55	0.50	0.051	2537.2	2459.0	2594.9	2600.4	2519.9	2491.0	2541.4	2623.8	2527.3	2576.7	2547.15	51.60
27	55	1.00	0.051	2329.1	2342.7	2455.1	2370.6	2426.1	2458.1	2382.5	2446.1	2388.2	2398.0	2399.65	45.75

Table 4.32: Data obtained from 27 trials on DeJong F2 function case 4

The significant effects of parameters, their interaction and the quadratic effects are shown in Table 4.33 for average and in Table 4.34 for standard deviation.

Design parameter		Effect
MP	mutation prob.	-2424.54
PS	population size	1035.92
PS*PS	population size (quad)	352.3
MP*MP	mutation prob. (quad)	-159.48
PS*MP	population size * mutation prob.	-67.55
CP*MP	crossover prob. * mutation prob.	-36.2
CP*CP	crossover prob. (quad.)	18.94

Table 4.33: Effects table for average case 4

Design parameter		Effect
PS*MP	population size * mutation prob.	272.6
PS	population size	-191.42
MP	mutation prob.	-182.69
MP*MP	mutation prob. (quad.)	-78.86
PS*PS	population size (quad.)	-36.015
CP*MP	crossover prob. * mutation prob.	30.455
CP	crossover prob.	-28.6
CP*CP	crossover prob. (quad.)	-26.9

Table 4.34: Effects table for standard deviation case 4

Response surface study

Based on 27 trials data in Table 4.32, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 2571.32 + 31.054PS - 24666.67MP - 0.17PS^2 - 134.43CP^2 \\ &\quad -20733.15MP^2 + 3625.19CP \times MP - 10.3PS \times MP \quad (4.24)\end{aligned}$$

The best fit surface through standard deviation is:

$$\begin{aligned}\log(\sigma) &= -5.2 - 0.016PS + 30.92CP - 23.89MP - 0.0002PS^2 - 19.24CP^2 \\ &\quad +298.578MP^2 - 46.1CP \times MP + 0.4338PS \times MP \quad (4.25)\end{aligned}$$

Analysis by direct observation

- Set PS = 100 , CP = 0.5 and MP = 0.001 in order to minimize variation.

Analysis by Variation transmission

Minimization of the equation for standard deviation of on-line performance obtained by equation 4.24 gives:

$$\sigma = 110.9 \text{ at } PS = 87, \text{ CP} = 1.0, \text{ and } MP = 0.001.$$

Putting the value PS = 87, CP =1.0 and MP = 0.1, in the equation 4.25 we get $\sigma = 31.28$. The value of standard deviation obtined by direct observation method is 5.62 . This means that result from variation transmission method is not good.

So with mean at PS = 100, CP = 0.5, and MP = 0.001 and standard deviation $\sigma_{PS} = 5$,$\sigma_{CP} = 0.05$ and $\sigma_{MP} = .005$ final capability study is done to verify the expected improvement.

4.2.5.1 Final Capability Study for Case 4

The data are given in Table 4.35. The control chart for average and standard deviation is also given in Figure 4.19 and Figure 4.20. We see that one point in standard deviation control chart is out of the control limit. Also the on-line performance obtained in capability study is much more compared to case 1 and case 2. So this means that with the tournament selection and uniform crossover it is not possible to make the process capable.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	3812.4	3823.2	3814.4	3810.5	3826.3	3815.2	3822.5	3808.3	3824.0	3814.2	3817.1	6.3
2	3841.9	3835.4	3830.6	3835.4	3845.2	3834.9	3844.2	3844.2	3837.9	3836.3	3839.2	4.2
3	3844.4	3830.3	3832.3	3835.2	3845.0	3804.4	3763.3	3826.8	3837.8	3836.3	3825.6	24.7
4	3835.7	3818.9	3827.0	3768.0	3844.3	3837.4	3846.4	3853.9	3847.1	3834.9	3831.3	24.5
5	3855.8	3838.2	3836.5	3833.6	3841.3	3852.5	3849.7	3845.8	3815.5	3835.3	3840.4	11.6
6	3771.6	3777.5	3773.9	3781.6	3776.1	3767.5	3803.1	3772.9	3775.9	3762.7	3776.3	10.8
7	3802.8	3829.3	3805.3	3804.1	3812.2	3813.6	3807.2	3792.3	3805.5	3790.5	3806.3	11.0
8	3855.4	3853.4	3836.6	3764.7	3862.9	3821.4	3782.9	3684.7	3532.1	3846.5	3784.1	104.3
9	3721.8	3713.1	3689.3	3723.5	3722.1	3716.7	3713.8	3705.7	3716.7	3729.3	3715.2	11.2
10	3857.9	3866.0	3857.8	3838.2	3866.0	3857.6	3860.7	3805.7	3864.7	3857.9	3853.3	18.5
11	3722.4	3705.3	3713.9	3723.4	3731.5	3713.1	3719.4	3730.6	3729.6	3689.0	3717.8	13.2
12	3738.5	3754.0	3738.1	3745.0	3739.7	3739.1	3733.4	3744.0	3753.0	3717.1	3740.2	10.4
13	3826.0	3827.4	3826.1	3841.7	3832.5	3834.8	3834.5	3840.5	3844.5	3831.1	3833.9	6.6
14	3850.6	3863.9	3854.5	3835.9	3860.2	3864.9	3865.1	3865.2	3864.0	3854.3	3857.9	9.4
15	3855.5	3838.4	3836.3	3814.4	3843.2	3853.8	3850.4	3843.6	3816.0	3834.5	3838.6	14.2
16	3773.5	3777.1	3771.0	3772.2	3774.2	3765.2	3800.9	3773.5	3774.4	3771.7	3775.4	9.5
17	3800.0	3828.8	3803.4	3801.9	3810.8	3807.9	3806.1	3791.0	3803.9	3790.4	3804.4	10.8
18	3808.8	3851.1	3846.4	3770.9	3863.3	3758.8	3861.9	3684.5	3856.0	3851.6	3815.3	59.6
19	3714.0	3709.9	3684.8	3725.3	3725.4	3707.0	3707.6	3701.4	3701.8	3726.4	3710.4	13.1
20	3857.9	3865.3	3857.8	3857.7	3861.1	3858.0	3785.7	3845.4	3864.8	3857.9	3851.2	23.6

Table 4.35: Final capability study for case 4 function F2

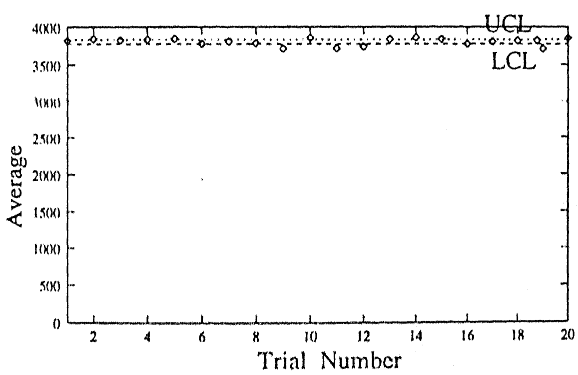

Figure 4.19: Control chart for average of on-line performance after variation reduction case 4 on function F2

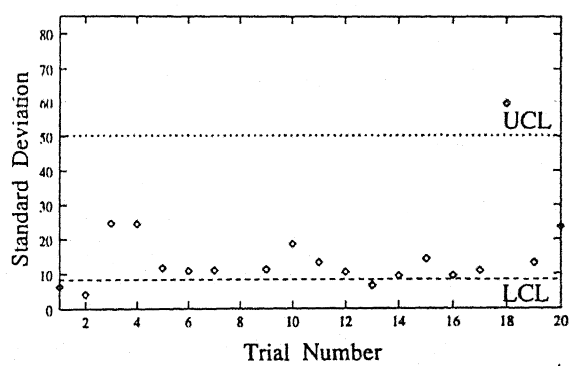

Figure 4.20: Control chart for standard deviation of on-line performance after variation reduction case 4 on function F2

4.2.6 Discussion of Result of Function F2

In the Table 4.36 the average and the standard deviation of the final capability study are shown for each case. The value of PS, CP and MP are given at which we have to fix it with standard deviation of 5, 0.05 and 0.005 respectively, for variation reduction.

Table 4.36: Result of function F2

		PS	CP	MP	average	std. deviation
CASE 1	Proportionate selection single point crossover	82	0.5	0.1	295.3	15.38
CASE 2	proportionate selection uniform crossover	55	0.5	0.1	295.88	15.23
CASE 3	tournament selection single point crossover	10	1	0.1	624.44	91.98
CASE 4	tournament selection uniform crossover	100	0.5	0.001	3801.69	30.08

We see that proportionate selection is always better for minimization of both on-line performance and variation in on-line performance. We see that if proportionate selection is the selection operator, both single point crossover and uniform crossover gives almost same value of average and standard deviation of capability study. So any of the crossover operator can be chosen with proportionate selection as selection operator. Also we have seen that in case 3 and case 4, when tournament selection is used it is not possible to make the process capable. So for this function proportionate selection is recommended.

In every case PS, CP and MP which has to be targeted, varies. Since we found that proportionate selection with any of the crossover operator is better, we can set PS at 55, CP at 0.5 and MP at 0.1 as mean with standard deviation of 5 for PS, 0.05 for CP and 0.005 for MP with uniform crossover as crossover operator. Otherwise we can set PS at 82, CP at 0.5 and MP at 0.1 as mean with standard deviation of 5 for PS, 0.05 for CP and 0.005 for MP with single point crossover as crossover operator.

4.3 Test Function 3 : Rastrigin's Function

4.3.1 Initial Capability Study

To check the need for variation reduction capability study is done. The on-line performance measured for last 50 % of the generation are shown in Table 4.37. The average and the standard deviation of each trial are also shown in Table 4.37.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	380.2	382.3	385.7	390.7	395.4	393.9	395.2	396.4	392.5	396.2	390.8	5.99
2	355.3	359.6	348.4	360.5	356.7	363.8	360.2	364.6	359.6	361.1	359.0	4.66
3	175.4	151.4	159.2	163.9	156.4	147.5	149.8	159.2	155.8	166.9	158.6	8.47
4	353.6	359.1	361.2	365.8	366.2	364.9	371.1	365.4	364.7	366.3	363.8	4.79
5	268.7	188.8	181.0	187.4	199.6	188.8	198.8	196.0	206.4	184.4	200.0	25.38
6	381.9	383.7	388.0	391.7	395.0	397.1	396.4	395.8	395.4	394.2	391.9	5.49
7	368.1	367.6	371.9	376.3	378.3	378.8	378.4	378.9	378.7	377.2	375.4	4.49
8	302.2	331.4	312.2	313.4	314.1	323.3	320.1	325.7	325.5	321.0	318.9	8.49
9	495.5	506.4	505.7	507.5	515.0	510.7	516.3	513.0	519.3	513.8	510.3	6.84
10	534.0	540.9	549.7	559.3	557.7	564.0	564.7	567.4	567.1	558.0	556.4	11.17
11	414.9	416.3	421.5	432.5	430.9	427.0	431.1	429.4	425.2	427.9	425.7	6.18
12	471.7	476.4	478.5	490.0	489.0	486.5	485.9	488.2	491.7	485.6	484.3	6.57
13	549.2	550.6	560.1	575.9	566.9	576.4	581.4	571.2	580.3	573.0	568.5	11.63
14	349.0	354.9	353.0	363.4	363.8	364.2	364.4	363.3	366.6	363.6	360.6	5.98
15	402.3	405.8	411.4	419.3	421.1	417.4	418.0	419.7	416.9	413.2	414.5	6.27
16	444.7	441.1	445.2	449.2	453.5	456.7	452.3	456.0	452.9	451.8	450.3	5.17
17	358.7	361.6	363.9	370.3	369.5	375.8	376.4	367.7	371.2	369.4	368.5	5.70
18	315.3	298.1	303.4	280.5	342.2	299.5	306.8	279.7	368.0	268.1	306.2	30.05
19	382.7	381.6	388.1	393.9	395.6	396.9	397.7	396.1	397.5	396.8	392.7	6.22
20	383.0	385.6	387.6	394.0	396.1	396.1	400.2	396.9	394.9	397.8	393.2	5.74

Table 4.37: Initial capability study for Rastrigin's function

These data are used to construct the average and standard deviation *control charts* shown in Figure 4.21 and Figure 4.22.

We can see from Figure 4.21 and Figure 4.22 that lot of points are going outside the control limit. So there is need to reduce the variation. For reducing the variation four cases as described in previous chapter is selected and for each case variation reduction technique is applied.

4.3.2 CASE 1 (Proportionate Selection and Single Point Crossover)

27 trial experiment data are shown in Table 4.38.

The significant effects of parameters, their interaction and the quadratic effects are

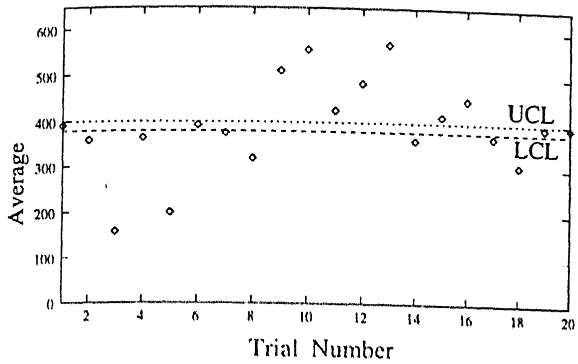

Figure 4.21: Control chart for average on-line performance before variation reduction on Rastrigin's function

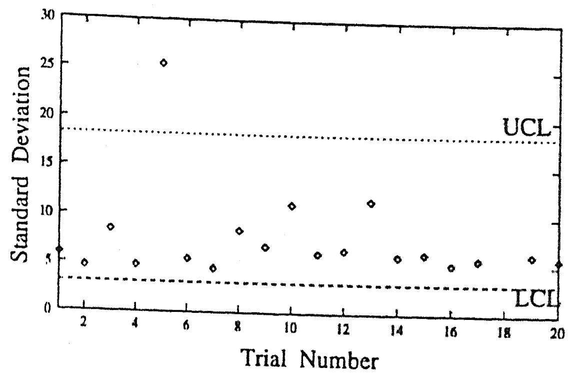

Figure 4.22: Control chart for standard deviation of on-line performance before variation reduction on Rastrigin's function

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	250	0.50	0.100	362.2	363.7	368.8	375.1	378.5	377.4	378.9	377.6	375.6	375.9	373.38	6.19
2	250	1.00	0.001	244.1	239.7	250.0	243.7	247.9	232.9	257.2	246.2	246.2	243.1	245.10	6.36
3	25	0.50	0.100	339.3	319.7	335.8	325.2	332.4	328.8	334.8	324.7	329.4	324.3	329.45	6.12
4	250	0.50	0.001	225.3	246.4	203.8	192.3	242.0	238.4	199.3	245.5	231.1	204.6	222.86	20.91
5	25	1.00	0.001	131.2	130.3	132.3	135.6	127.8	136.8	121.6	134.9	135.7	135.5	132.18	4.72
6	25	1.00	0.100	306.3	322.2	313.1	320.4	320.8	336.3	325.9	311.9	320.8	323.0	320.07	8.26
7	250	1.00	0.100	360.8	361.9	367.8	374.3	377.8	374.2	375.8	375.5	377.4	374.7	372.02	6.28
8	25	0.50	0.001	131.0	133.5	136.4	138.7	140.3	139.6	139.5	140.6	138.0	137.7	137.54	3.12
9	137	0.75	0.051	332.9	331.0	342.8	349.9	351.7	348.1	350.6	350.8	343.6	350.9	345.22	7.64
10	137	0.50	0.001	160.2	140.3	139.7	140.7	147.4	147.7	157.6	149.9	146.8	153.9	148.39	7.15
11	137	0.50	0.100	358.5	357.9	362.6	366.5	370.6	376.3	367.5	374.4	371.4	370.9	367.66	6.32
12	137	1.00	0.001	172.1	132.2	138.8	137.1	136.4	140.0	141.8	138.2	139.5	141.2	141.72	11.01
13	137	1.00	0.100	359.3	361.9	365.4	365.9	372.7	372.5	372.8	370.6	371.1	375.0	368.70	5.26
14	25	0.75	0.001	188.0	160.4	143.6	125.4	116.2	120.8	115.2	114.8	114.2	116.7	131.53	24.98
15	25	0.75	0.100	320.0	328.6	325.3	330.7	326.9	334.1	335.0	330.3	330.2	335.5	329.65	4.76
16	250	0.75	0.001	227.3	242.1	234.9	243.3	240.4	254.3	226.9	237.4	220.1	241.4	230.81	9.91
17	250	0.75	0.100	361.3	363.5	368.2	373.7	374.2	370.3	374.6	374.1	377.9	375.9	371.96	5.67
18	25	0.50	0.051	321.6	274.0	337.0	299.4	326.5	278.5	314.8	305.0	282.8	277.5	301.71	22.84
19	25	1.00	0.051	272.1	278.3	277.9	290.5	327.0	286.9	295.9	286.6	283.9	319.7	291.88	17.98
20	250	0.50	0.051	345.9	348.7	336.4	366.8	361.7	362.5	365.8	358.6	362.8	360.8	357.00	9.96
21	250	1.00	0.051	349.3	352.7	356.8	366.0	363.8	357.8	361.7	356.0	361.5	358.0	358.36	5.07
22	137	0.75	0.001	180.5	127.5	137.0	137.1	142.5	138.6	137.1	144.9	142.8	145.6	143.35	14.05
23	137	0.75	0.100	359.0	352.2	360.5	370.1	366.9	368.4	370.8	370.7	370.3	370.2	365.91	6.45
24	25	0.75	0.051	282.7	283.6	289.9	321.5	345.1	325.9	317.3	312.6	292.0	327.3	309.79	21.46
25	250	0.75	0.051	348.8	350.9	355.1	357.4	364.8	363.0	363.1	364.6	361.2	361.8	359.07	5.77
26	137	0.50	0.051	328.6	339.4	339.0	354.1	354.5	348.8	348.8	344.3	344.7	350.5	345.28	7.94
27	137	1.00	0.051	341.2	338.4	346.8	346.7	353.3	360.4	357.8	341.7	354.1	347.3	348.77	7.37

Table 4.38: Data obtained from 27 trials on Rastrigin's function case 1

shown in Table 4.39 for average and in Table 4.40 for standard deviation.

Design parameter		Effect
MP	mutation prob.	184.37
MP*MP	mutation prob. (quad)	71.992
PS	population size	68.08
PS*MP	population size * mutation prob.	-27.55
PS*CP	population size * crossover prob.	7.8
CP*MP	crossover prob.* mutation prob.	-3.32

Table 4.39: Effects table for average case 1

Design parameter		Effect
MP	mutation prob.	-5.21
PS	population size	-4.23
PS*CP	population size*crossover prob.	-3.04
MP*MP	mutation prob.(quad.)	3.02
PS*PS	population size (quad.)	-2.441
CP*CP	crossover prob. (quad.)	2.14
CP	crossover prob.	-2.02
CP*MP	crossover prob.* mutation prob.	1.71

Table 4.40: Effects table for standard deviation case 1

Response surface study

Based on 27 trials data in Table 4.38, best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 130.16 + 0.4088PS + 3036.84MP - 10881.16MP^2 \\ &\quad -0.01PS \times CP + 200.76CP \times MP - 1.897PS \times MP \qquad (4.26)\end{aligned}$$

The best fit surface through standard deviation is:

$$\begin{aligned}\log(\sigma) &= 4.926 - 0.0217PS + 1.55CP - 2.11MP + 0.000003PS^2 - 1.446CP^2 \\ &\quad -81.18MP^2 - 0.000058PS \times CP + 7.622CP \times MP \qquad (4.27)\end{aligned}$$

Analysis by direct observation

The result from this analysis can be concluded as:

- Set PS = 250, CP = 0.5 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of equation for stadard deviation, obtained from equation 4.26 of on-line performance gives:

$$\sigma = 2.83 \text{ at } PS = 250,\ CP = 0.5, \text{ and } MP = 0.1.$$

This means that result from analysis of variation transmission method and by direct observation method lead to same value of PS, CP and MP. Now the transmitted variation of each parameter was found and given as:

$\sigma_{tPS} = 1.063$, $\sigma_{tCP} = 0.84$ and $\sigma_{tMP} = 2.46$

This shows that more control on variation of MP is required than PS and CP. So there is need to reduce the variation of MP i.e. MP is a VIP.

So with mean at PS = 250, CP = 0.5 and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = .05$, and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.3.2.1 Final Capability Study for Case 1

The data are given in Table 4.41. The control chart for average and the standard deviation is also given in Figure 4.23 and Figure 4.24. We see that process is capable as all points are within the control limits. Now we try next case.

4.3.3 CASE 2 (Proportionate Selection and Uniform Crossover)

27 trial data are shown in Table 4.42.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	359.4	362.8	367.8	372.4	378.3	374.1	376.1	376.4	377.8	376.3	372.1	6.61
2	361.5	364.4	368.2	372.9	371.8	372.3	374.7	372.0	375.2	373.1	370.6	4.50
3	359.9	361.0	368.5	371.8	373.4	377.4	375.2	378.1	375.9	371.1	371.2	6.38
4	362.2	361.3	364.6	371.4	376.6	373.2	379.4	373.9	376.5	370.9	371.0	6.31
5	361.0	359.1	367.1	373.6	371.4	375.5	374.4	378.5	372.1	374.7	370.7	6.38
6	362.2	363.6	369.0	375.6	379.6	375.7	377.0	377.2	380.5	374.8	373.5	6.40
7	357.9	363.8	368.0	375.8	378.1	375.4	374.9	375.1	372.9	373.7	371.6	6.37
8	356.3	362.8	365.8	373.9	367.7	372.7	374.9	371.1	372.8	374.9	369.3	6.11
9	361.4	367.2	367.3	373.9	374.2	377.4	378.4	377.1	376.4	377.4	373.1	5.78
10	362.6	361.3	365.9	373.7	372.3	374.4	375.5	373.5	378.1	375.5	371.3	5.86
11	364.1	364.4	370.5	377.5	380.3	378.0	370.5	378.6	377.6	379.0	374.6	6.06
12	363.3	364.8	367.3	375.9	378.1	374.0	375.8	378.8	378.1	375.9	373.2	5.82
13	364.6	363.7	370.7	377.2	374.8	377.5	373.8	376.0	377.6	379.1	373.5	5.47
14	356.7	360.5	365.7	372.5	373.1	375.5	376.5	371.8	372.4	369.8	369.4	6.49
15	363.5	363.0	366.3	374.6	371.9	375.6	372.6	375.6	372.3	372.3	370.8	4.75
16	363.1	360.8	368.5	375.2	377.1	375.0	374.6	375.7	375.3	376.5	372.2	5.90
17	361.8	359.7	368.4	373.7	374.1	373.7	377.7	375.3	376.2	374.9	371.5	6.20
18	359.6	360.5	365.5	375.3	372.2	376.7	375.3	370.9	375.3	375.7	370.7	6.50
19	362.9	361.1	368.0	375.5	376.0	376.8	377.1	378.8	374.2	377.0	372.7	6.38
20	362.1	359.4	368.7	374.4	376.3	375.9	372.3	376.6	376.0	376.7	371.8	6.38

Table 4.41: Final capability study for case 1 Rastrigin's function

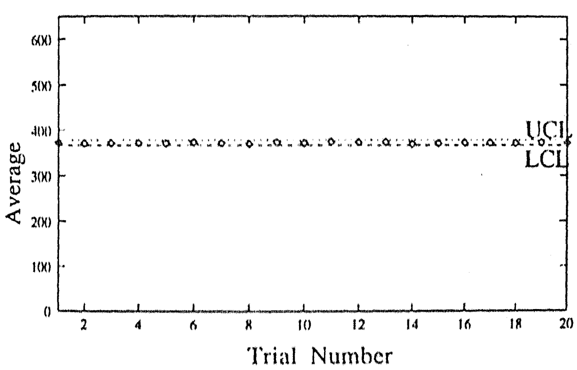

Figure 4.23: Control chart for average of on-line performance after variation reduction for case 1 on Rastrigin's function

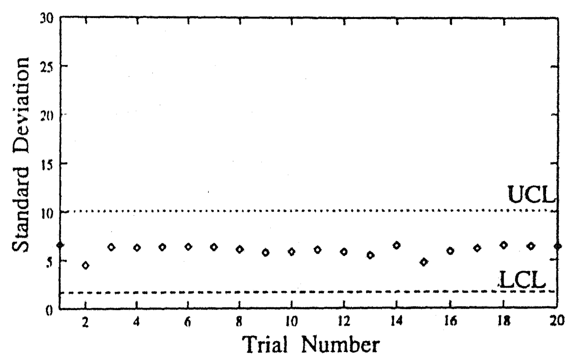

Figure 4.24: Control chart for standard deviation of on-line performance after variation reduction for case 1 on Rastrigin's function

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	250	0.50	0.100	365.9	369.8	375.2	379.7	380.7	381.6	379.6	381.6	381.8	381.9	377.78	5.66
2	250	1.00	0.001	350.1	354.7	358.2	370.7	368.8	363.3	368.7	359.1	367.7	367.1	362.84	6.98
3	25	0.50	0.100	325.1	334.2	325.6	334.0	344.0	339.1	338.7	341.0	313.3	338.4	333.33	9.37
4	250	0.50	0.001	342.6	302.7	344.1	344.5	341.2	356.7	347.9	354.4	347.9	358.9	344.08	15.76
5	25	1.00	0.001	138.8	141.9	147.1	140.8	141.4	135.0	147.2	147.5	146.1	148.3	143.42	4.48
6	25	1.00	0.100	347.8	343.1	347.8	340.3	351.2	349.2	355.9	344.2	354.0	348.7	348.21	4.81
7	250	1.00	0.100	368.2	370.2	376.9	382.1	382.9	385.0	383.4	383.1	385.3	382.5	379.96	6.13
8	25	0.50	0.001	145.3	146.0	147.7	149.7	147.3	149.4	145.6	152.3	142.3	150.8	147.63	2.99
9	137	0.75	0.051	357.8	359.5	365.6	371.7	373.5	372.6	370.8	372.3	376.6	372.2	369.25	6.24
10	137	0.50	0.001	298.7	276.5	303.4	273.3	286.2	258.2	281.7	287.6	294.7	308.9	286.93	15.29
11	137	0.50	0.100	361.6	365.5	369.0	370.2	375.7	374.6	374.0	376.1	376.7	378.6	372.20	5.48
12	137	1.00	0.001	309.9	299.6	281.9	309.8	292.9	302.3	303.9	300.6	322.1	306.0	302.91	10.72
13	137	1.00	0.100	365.6	368.0	371.9	377.6	380.5	380.8	377.6	379.2	378.9	380.9	376.12	5.58
14	25	0.75	0.001	197.4	139.2	154.8	145.9	162.7	158.9	160.5	150.8	148.1	140.0	155.82	16.73
15	25	0.75	0.100	344.7	331.5	320.5	334.8	341.6	339.5	332.3	329.6	318.0	326.9	331.96	8.67
16	250	0.75	0.001	340.4	337.0	356.6	356.0	359.7	356.5	369.7	360.9	357.5	359.0	355.34	9.63
17	250	0.75	0.100	368.5	366.2	374.1	381.3	380.3	382.1	381.8	382.5	382.5	382.8	378.21	6.29
18	25	0.50	0.051	292.2	275.7	297.8	289.3	289.6	285.6	298.1	304.3	273.1	298.8	290.45	10.12
19	25	1.00	0.051	312.6	302.4	344.3	324.3	325.1	320.3	325.9	320.5	318.7	325.3	321.94	10.68
20	250	0.50	0.051	360.1	364.3	371.0	372.8	373.8	374.6	376.9	376.9	374.1	374.6	371.91	5.48
21	250	1.00	0.051	366.9	369.8	373.8	379.4	382.5	382.2	381.6	382.7	381.3	380.4	378.06	5.76
22	137	0.75	0.001	278.4	307.4	301.8	304.9	307.0	308.9	280.6	332.3	295.0	310.2	302.65	15.48
23	137	0.75	0.100	368.0	366.7	371.0	375.3	378.5	377.9	379.9	379.2	377.9	378.4	375.28	4.89
24	25	0.75	0.051	289.5	286.2	321.2	322.6	298.7	314.3	318.3	309.2	290.1	307.0	305.71	13.74
25	250	0.75	0.051	364.7	367.0	371.0	377.8	378.1	381.4	379.9	379.4	379.7	380.1	375.92	6.02
26	137	0.50	0.051	353.8	358.8	355.4	365.0	368.4	367.2	364.4	370.7	357.1	368.1	362.90	6.10
27	137	1.00	0.051	363.4	362.3	367.2	375.0	376.5	378.1	376.7	376.4	375.9	377.1	372.86	6.07

Table 4.42: Data obtained from 27 trials on Rastrigin's function case 2

The significant effects of parameters, their interaction and the quadratic effects are shown in Table 4.43 for average and in Table 4.44 for standard deviation.

Design parameter		Effect
PS	population size	105.1
MP	mutation prob.	96.82
PS*MP	population size * mutation prob.	-82.15
MP*MP	mutation prob. (quad)	34.62
PS*PS	population size (quad)	29.98
CP	crossover prob.	-11.01

Table 4.43: Effects table for average case 2

Response surface study

Based on 27 trials data in Table 4.42, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned} \text{on-line performance} &= 118.2 + 1.49PS + 23.35CP + 2259.16MP - 0.00239PS^2 \\ &\quad -2918.11MP^2 - 7.235PS \times MP \end{aligned} \tag{4.28}$$

Design parameter		Effect
MP	mutation prob.	-4.57
PS*MP	population size * mutation prob.	-2.15
CP*CP	crossover prob.(quad.)	2.1
CP	crossover prob.	-1.67
PS	population size	-1.54
CP*MP	crossover prob. * mutation prob.	1.31

Table 4.44: Effects table for standard deviation case 2

The best fit surface through standard deviation is:

$$\log(\sigma) = -0.4438 + 0.0011PS + 7.0CP + 7.569MP - 4.53CP^2 - 9.55CP \times MP - 0.0336PS \times MP \quad (4.29)$$

Analysis by direct observation

- Set PS = 250 , CP = 1 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimizing the equation for standard deviation obtined from equation 4.28 gives:

$$\sigma = 1.45 \text{ at } PS = 250,\ CP = 0.68 \text{ and } MP = 0.055.$$

The standard deviation from equation 4.29 will be 8.114 at PS = 250, CP = 0.68 and MP = 0.055. The standard deviation obtained from equation 4.29 by direct observation-method is 3.61. This means that the result from Variation transmission method method is not good.

So with mean at PS = 250, CP = 1 and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = .05$, and $\sigma_{MP} = .005$ final capability study is done to verify the expected improvement.

4.3.3.1 Final Capability Study for case 2

The data are given in Table 4.45. The control chart for average and standard deviation is also given in Figure 4.25 and Figure 4.26. We see that all points are within the control limits. Hence the process is capable. Now we try for next case.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	371.1	371.1	374.7	382.8	383.7	383.3	384.1	383.6	383.0	384.2	380.2	5.54
2	370.6	371.2	374.7	381.5	383.2	383.7	383.1	384.5	383.3	382.9	379.9	5.48
3	372.2	371.1	374.4	380.0	382.3	382.4	382.4	382.9	382.4	381.9	379.2	4.70
4	370.0	370.2	375.8	380.0	381.3	381.7	383.5	382.5	384.7	382.9	379.2	5.40
5	369.0	371.2	375.8	380.5	383.6	382.5	382.5	384.0	384.9	383.2	379.7	5.70
6	368.0	371.4	375.0	379.9	383.1	383.9	383.7	383.2	385.0	383.6	379.7	6.04
7	369.8	369.9	375.1	380.0	384.1	384.2	382.9	384.5	383.4	384.7	379.9	6.03
8	368.8	369.5	373.8	381.7	382.3	384.0	382.9	383.3	382.2	384.3	379.3	6.13
9	368.8	371.0	375.6	382.9	383.5	383.2	382.9	385.0	382.6	383.6	379.9	5.87
10	370.2	369.8	373.5	381.8	383.6	385.0	383.2	384.3	382.9	383.6	379.8	6.08
11	370.1	370.0	374.2	380.3	385.1	386.3	384.7	385.0	382.5	383.7	380.2	6.37
12	367.6	370.8	374.2	382.8	383.9	384.1	385.4	382.9	383.7	383.3	379.9	6.44
13	369.5	370.8	376.0	383.3	383.6	383.9	383.7	381.1	383.1	385.4	380.0	5.81
14	369.1	368.9	375.1	381.1	385.3	384.7	382.3	382.4	383.6	384.4	379.7	6.32
15	369.3	369.5	376.2	380.1	382.1	384.2	382.6	381.8	383.5	380.6	379.0	5.50
16	369.4	369.6	374.4	382.9	382.9	383.4	382.5	384.2	384.9	383.7	379.8	6.15
17	367.5	369.3	373.3	381.1	383.8	383.3	384.2	383.3	383.3	382.7	379.2	6.50
18	367.3	369.7	373.4	380.4	381.9	383.4	381.9	382.1	384.0	381.6	378.6	6.07
19	370.3	369.9	375.8	383.5	382.2	384.4	382.9	381.3	380.6	384.8	379.6	5.61
20	368.5	371.6	375.4	380.4	383.6	383.7	384.6	382.9	384.4	382.1	379.7	5.79

Table 4.45: Final capability study for case 2 Rastrigin's function

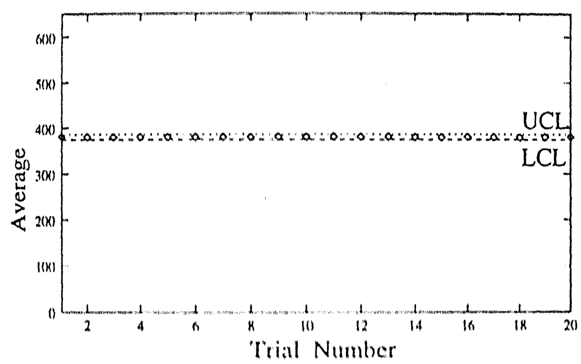

Figure 4.25: Control chart for average of on-line performance after variation reduction case 2 on Rastrigin's function

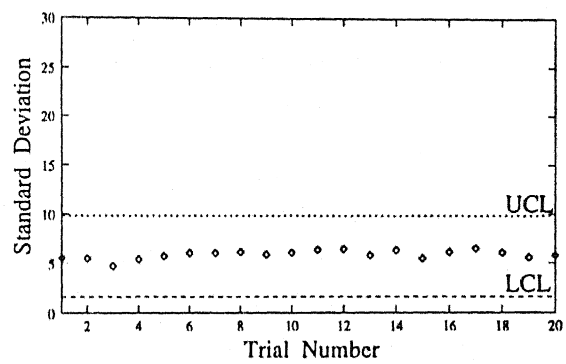

Figure 4.26: Control chart for standard deviation of on-line performance after variation reduction case 2 on Rastrigin's function

4.3.4 CASE 3 (Tournament Selection and Single Point Crossover)

27 trial experiment data are shown in Table 4.46.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	250	0.50	0.100	404.2	406.9	408.9	416.8	418.2	417.7	418.2	419.0	418.5	421.0	414.93	5.92
2	250	1.00	0.001	721.0	705.0	720.4	719.5	745.6	741.2	736.5	739.2	742.6	733.4	730.45	13.23
3	25	0.50	0.100	379.2	382.1	387.1	387.3	389.2	385.8	390.8	388.2	394.3	385.8	386.98	4.24
4	250	0.50	0.001	673.4	693.5	692.7	726.3	721.0	706.2	716.2	711.2	722.2	708.2	707.09	16.41
5	25	1.00	0.001	677.9	653.7	677.7	660.9	695.2	599.1	609.0	633.0	673.9	621.6	650.22	32.77
6	25	1.00	0.100	383.4	381.6	391.1	388.8	391.2	388.5	391.5	385.6	391.5	388.5	388.17	3.54
7	250	1.00	0.100	404.4	405.6	412.6	417.8	419.4	416.6	420.6	418.5	421.4	416.3	415.32	5.97
8	25	0.50	0.001	660.2	639.5	663.1	654.3	662.6	594.4	578.9	606.3	676.1	616.7	635.22	33.70
9	137	0.75	0.051	443.3	446.8	443.9	454.7	460.5	456.9	455.9	460.5	459.8	454.0	453.63	6.66
10	137	0.50	0.001	733.0	740.5	705.2	746.9	769.5	742.5	756.5	756.2	766.3	758.4	747.48	18.80
11	137	0.50	0.100	401.7	401.8	407.5	411.2	421.0	416.3	413.9	413.7	414.5	415.1	411.67	6.24
12	137	1.00	0.001	761.1	759.0	762.8	757.8	783.5	783.2	775.2	789.1	763.9	755.7	769.14	12.40
13	137	1.00	0.100	402.4	400.0	405.4	410.9	415.8	416.1	413.5	416.8	418.4	416.5	411.58	6.64
14	25	0.75	0.001	644.2	658.4	657.7	663.6	648.4	631.6	623.7	647.8	650.0	602.3	642.77	18.68
15	25	0.75	0.100	381.0	378.3	385.3	389.4	389.3	387.7	384.4	387.6	392.0	388.5	386.35	4.16
16	250	0.75	0.001	698.2	703.0	712.8	735.2	716.9	721.6	736.9	714.4	739.2	730.5	720.86	14.33
17	250	0.75	0.100	405.3	404.4	411.1	417.2	417.9	419.5	419.5	417.0	417.4	416.9	414.62	5.65
18	25	0.50	0.051	413.5	417.4	421.1	423.6	425.0	419.2	420.5	419.4	424.2	416.7	420.05	3.63
19	25	1.00	0.051	406.5	411.6	420.1	421.7	439.9	418.3	419.1	420.0	424.8	419.2	420.12	8.68
20	250	0.50	0.051	439.1	448.6	450.9	462.4	460.9	458.0	459.7	460.8	460.8	460.8	456.19	7.60
21	250	1.00	0.051	443.7	444.6	451.4	460.8	464.2	460.8	463.6	460.5	458.7	461.0	456.93	7.56
22	137	0.75	0.001	746.3	748.1	770.3	757.0	783.6	756.1	766.6	754.7	775.8	769.7	762.83	12.25
23	137	0.75	0.100	402.5	400.1	404.7	411.0	415.1	414.8	412.4	418.3	414.2	416.2	410.92	6.29
24	25	0.75	0.051	406.3	405.3	419.3	419.8	423.9	422.2	421.7	416.7	423.4	418.6	417.72	6.66
25	250	0.75	0.051	445.8	447.6	452.2	455.1	460.1	461.9	461.8	460.8	463.7	458.3	456.75	6.29
26	137	0.50	0.051	444.5	436.5	447.3	451.4	457.5	458.4	455.8	459.4	457.4	451.4	451.98	7.38
27	137	1.00	0.051	442.2	442.7	448.1	455.4	458.6	453.3	455.7	455.4	461.2	456.6	452.93	6.47

Table 4.46: Data obtained from 27 trials on Rastrigin's function case 3

The significant effects of the parameters, their interaction and the quadratic effects are shown in Table 4.47 for average and in Table 4.48 for standard deviation.

Design parameter		Effect
MP	mutation prob.	-302.83
MP*MP	mutation prob. (quad)	-113.003
PS	population size	47.28
PS*PS	population size (quad)	34.64
PS*MP	population size * mutation prob.	-24.46
CP*MP	crossover prob. * mutation prob.	-9.75
CP	crossover prob.	7.029

Table 4.47: Effects table for average case 3

Response surface study

Based on 27 trials data in Table 4.46, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 543.6 + 1.13PS + 85.36CP - 3558.68MP - 0.00274PS^2 \\ &+20962.92MP^2 - 1447.79CP \times MP - 3.35PS \times MP \quad (4.30)\end{aligned}$$

Table 4.48: Effects table for standard deviation case 3

Design parameter		Effect
MP	mutation prob.	-13.78
PS*MP	population size * mutation prob.	7.797
MP*MP	mutation prob. (quad)	-5.52
PS	population size	-3.67
CP*CP	crossover prob.(quad.)	-2.18
PS*PS	population size (quad.)	-1.82
CP*MP	crossover prob. * mutation prob.	1.71

The best fit surface through standard deviation is:

$$\begin{aligned}\log(\sigma) &= 3.155 - 0.00242PS - 23.96MP - 0.061CP^2 + 41.42MP^2 \\ &+2.172CP \times MP + 0.043PS \times MP \end{aligned} \tag{4.31}$$

Analysis by direct observation

- Set PS = 25 , CP = 0.5 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of equation for standard devaition obtained from equation 4.30 gives:

$$\sigma = 3.95 \text{ at } PS = 98, \text{ CP} = 0.5 \text{ and } MP = 0.1.$$

Putting the value PS = 98, CP = 0.5 and MP = 0.1 in the equation 4.31 of standard deviation of on-line performance we get $\sigma = 4.29$. The value of standard deviation obtained by direct observation method is 3.72. This means that the result from variation transmission method is good.

The transmitted variation of the parameter is:

$$\sigma_{tPS} = 1.3, \sigma_{tCP} = 4.25, \sigma_{tMP} = 1.69$$

We see that transmitted variation by CP is largest, hence CP is a VIP. So with mean

at PS = 98, CP = 0.5, and MP = 0.1 and standard deviation $\sigma_{PS} = 5$ and $\sigma_{MP} = .005$ final capability study is done to verify the expected improvement.

4.3.4.1 Final Capability Study for Case 3

The data are given in Table 4.49. The control chart for average and standard deviation is also given in Figure 4.27 and Figure 4.28 . We see that process is capable as all points fall within the control limits. Now we try for next case.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	400.5	400.1	407.7	411.4	410.9	413.5	413.8	414.1	413.9	411.1	409.7	5.33
2	400.0	402.9	406.4	414.2	416.2	415.4	413.5	414.0	417.4	412.3	411.2	5.96
3	396.3	402.5	407.5	412.5	412.7	410.5	417.9	412.4	412.9	411.3	409.7	6.15
4	401.6	398.5	404.3	412.5	412.0	410.7	413.1	411.0	410.1	411.4	408.5	5.14
5	398.6	403.0	410.9	411.4	415.3	410.8	413.0	417.6	410.8	410.5	410.2	5.56
6	396.3	399.7	404.5	411.9	411.2	410.9	415.6	412.1	412.5	411.9	408.7	6.31
7	401.7	400.7	406.6	412.5	414.7	410.7	421.5	413.3	409.7	408.9	410.0	6.15
8	395.1	396.3	400.7	410.1	408.1	411.2	408.6	412.9	413.0	412.4	406.8	6.88
9	402.2	402.8	409.4	411.5	416.6	413.3	413.9	415.7	414.4	414.0	411.4	5.10
10	397.1	401.8	403.6	408.6	406.2	412.1	412.7	411.1	411.4	413.1	407.8	5.43
11	397.3	400.8	403.0	410.9	413.3	412.9	414.1	410.2	411.6	411.2	408.5	5.90
12	396.9	398.9	404.7	411.1	414.4	414.6	412.8	412.0	416.1	411.4	409.3	6.75
13	401.2	400.7	408.0	414.5	413.5	414.9	415.1	416.2	414.8	414.4	411.3	5.92
14	399.7	400.2	404.8	410.6	413.9	416.5	413.7	412.0	418.7	411.4	410.1	6.50
15	395.0	399.4	401.0	410.8	411.5	412.6	408.8	412.3	412.4	410.4	407.4	6.44
16	399.2	404.1	403.4	409.6	410.0	413.2	409.9	411.9	412.8	413.9	408.8	4.90
17	399.4	401.3	408.0	410.5	413.5	414.9	411.2	418.5	413.7	411.7	410.3	5.94
18	397.6	398.9	401.6	407.8	411.0	410.2	409.7	408.8	409.0	411.5	406.6	5.20
19	402.1	402.6	406.3	411.2	413.8	417.1	414.3	413.9	413.5	415.3	411.0	5.38
20	400.5	400.6	409.4	413.8	415.0	416.8	416.8	417.0	416.3	416.9	412.3	6.62

Table 4.49: Final capability study for case 3 Rastrigin's function

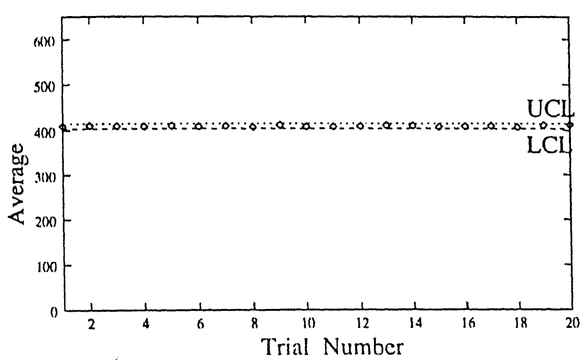

Figure 4.27: Control chart for average of on-line performance after variation reduction case 3 on Rastrigin's function

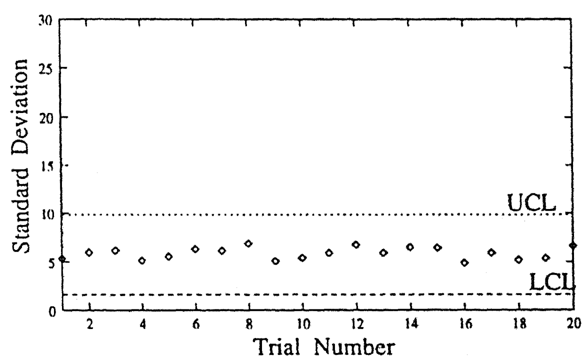

Figure 4.28: Control chart for standard deviation of on-line performance after variation reduction case 3 on Rastrigin's function

4.3.5 CASE 4 (Tournament Selection and Uniform Crossover)

27 trials experiment data are shown in Table 4.50.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	250	0.50	0.100	390.1	391.7	395.4	403.2	401.4	405.0	404.2	403.2	403.4	404.0	400.17	5.57
2	250	1.00	0.001	460.4	467.5	452.6	516.0	486.9	503.3	506.1	483.1	493.0	499.9	486.89	21.00
3	25	0.50	0.100	372.7	369.7	372.4	377.1	376.7	377.3	375.3	377.4	380.7	376.9	375.62	3.17
4	250	0.50	0.001	489.1	506.1	507.2	484.9	520.6	509.8	556.8	487.2	478.9	511.3	505.19	22.74
5	25	1.00	0.001	601.9	581.8	626.6	663.5	650.9	636.2	635.0	641.2	636.0	617.2	629.03	23.72
6	25	1.00	0.100	366.0	367.4	371.2	370.7	371.2	368.3	368.0	369.3	374.3	372.0	369.83	2.50
7	250	1.00	0.100	380.5	380.6	385.3	393.1	394.1	392.9	393.9	393.1	393.3	393.8	390.06	5.64
8	25	0.50	0.001	543.0	605.3	643.7	646.4	633.9	661.5	627.8	634.8	676.0	627.0	629.93	36.19
9	137	0.75	0.051	388.7	391.7	397.2	402.4	406.5	404.2	407.5	405.2	406.2	407.4	401.69	6.82
10	137	0.50	0.001	633.8	658.2	673.7	681.0	706.6	742.8	670.9	718.7	665.2	702.2	685.30	32.18
11	137	0.50	0.100	386.2	389.0	390.7	398.5	399.8	399.1	399.4	400.2	399.6	401.2	396.37	5.49
12	137	1.00	0.001	641.8	654.7	662.0	688.0	696.0	692.5	657.5	683.5	682.6	664.5	672.32	18.49
13	137	1.00	0.100	377.8	376.8	383.9	391.6	390.9	393.1	391.2	391.1	391.4	391.6	387.94	6.12
14	25	0.75	0.001	657.9	648.9	655.1	643.5	651.9	615.0	611.9	625.4	646.3	610.3	636.62	18.89
15	25	0.75	0.100	369.0	367.9	373.6	374.3	372.7	371.3	373.4	373.9	381.6	371.1	372.88	3.72
16	250	0.75	0.001	491.1	462.6	471.7	474.4	480.5	507.7	491.0	489.0	499.3	532.1	489.95	20.00
17	250	0.75	0.100	386.3	382.8	388.2	398.7	397.9	398.0	398.0	397.7	398.9	396.0	394.25	6.04
18	25	0.50	0.051	383.4	388.1	396.6	391.2	397.5	394.5	398.9	396.3	399.9	394.4	394.08	5.15
19	25	1.00	0.051	373.0	380.9	386.5	389.9	384.1	386.9	387.6	388.5	394.2	384.2	385.58	5.69
20	250	0.50	0.051	404.6	405.1	405.1	415.5	414.4	418.9	419.9	415.2	416.0	417.9	413.27	5.99
21	250	1.00	0.051	386.3	385.5	393.0	399.5	401.2	402.3	399.6	401.6	401.9	398.1	396.90	6.39
22	137	0.75	0.001	656.6	662.5	670.4	690.4	675.3	669.7	667.8	687.7	660.6	669.8	672.28	12.28
23	137	0.75	0.100	383.6	381.3	387.5	394.2	393.0	394.6	394.0	395.0	397.1	393.0	391.51	5.44
24	25	0.75	0.051	383.1	381.0	387.7	393.0	393.5	387.6	389.6	390.4	400.5	386.5	389.29	5.57
25	250	0.75	0.051	392.9	392.7	399.1	408.0	406.7	406.4	409.1	408.0	408.8	407.4	403.90	6.48
26	137	0.50	0.051	403.0	400.0	407.3	412.1	414.8	415.8	414.1	414.1	416.1	411.4	410.85	5.61
27	137	1.00	0.051	387.0	387.2	392.4	396.2	398.4	399.7	401.9	402.8	402.1	398.9	396.65	5.91

Table 4.50: Data obtained from 27 trials on Rastrigin's function case 4

The significant effects of the parameters, their interactions and the quadratic effects are shown in Table 4.51 for average and in Table 4.52 for standard deviation.

Design parameter		Effect
MP	mutation prob.	-214.32
MP*MP	mutation prob. (quad)	-94.54
PS*MP	population size * mutation prob.	79.95
PS*PS	population size (quad)	42.576
PS	population size	-33.58
CP	crossover prob.	-10.62
PS*CP	population size * crossover prob.	-4.933

Table 4.51: Effects table for average case 4

Response surface study

Based on 27 trials data in Table 4.50, the best surface through the average is fitted. The

Table 4.52: Effects table for standard deviation case 4

Design parameter		Effect
MP	mutation prob.	-17.98
MP*MP	mutation prob. (quad)	-7.887
CP*MP	crossover prob. * mutation prob.	4.656
PS*MP	population size * mutation prob.	3.82
CP	crossover prob.	-2.96
CP*CP	crossover prob.(quad.)	-2.61
PS*CP	population size * crossover prob.	1.887

resulting equation is :

$$\text{on-line performance} = 626.28 + 0.4746PS - 18.37MP - 4424.5MP - 0.003344PS^2$$
$$+14031.33MP^2 - 0.041PS \times CP + 6.45PS \times MP \qquad (4.32)$$

The best fit surface through standard deviation is:

$$\log(\sigma) = 6.127 - 7.64CP - 38.98MP + 4.383CP^2 + 53.92MP^2 - 0.0007PS \times CP$$
$$+18.36CP \times MP + 0.03PS \times MP \qquad (4.33)$$

Analysis by direct observation

- Set PS = 43 , CP = 0.66 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of the equation for standard deviation obtained from equation 4.32 gives:

$$\sigma = 2.47 \text{ at } PS = 204, \text{ CP} = 0.54, \text{ and } MP = 0.1$$

Putting the value PS = 204, CP =0.54 and MP = 0.1 in the equation 4.33 of standard deviation of on-line performance we get $\sigma = 4.27$.The value of standard deviation obtained by direct observation method is 2.6. This means that result from variation transmission method is not good.

So with mean at PS = 43, CP = 0.66, and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$ and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.3.5.1 Final Capability Study for Case 4

The data are given in Table 4.53. The control chart for average and standard deviation is also given in Figure 4.29 and Figure 4.30 . We see that process is capable as all points are within the control limits except one point in Figure 4.30, which is slightly above the UCL.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	372.8	374.2	380.4	379.3	382.2	382.6	383.2	380.2	382.3	380.6	379.8	3.54
2	370.4	374.8	378.9	378.1	380.9	382.5	383.7	380.4	378.8	378.5	379.3	2.70
3	371.9	367.4	370.4	382.4	393.2	381.8	385.1	394.6	371.8	386.9	381.1	9.20
4	369.2	371.1	380.0	377.0	373.5	371.7	378.4	373.4	378.8	376.0	374.9	3.66
5	370.3	368.2	368.1	370.4	378.2	374.3	377.2	381.3	376.8	375.7	375.0	4.65
6	371.0	372.5	376.2	370.4	378.0	379.6	378.4	376.7	383.0	377.2	377.2	3.48
7	371.9	370.3	381.7	388.7	388.5	384.8	387.9	372.8	387.8	386.0	382.0	7.49
8	374.2	373.4	381.9	386.8	381.2	371.2	387.9	367.5	385.2	386.8	379.6	7.42
9	370.0	374.8	376.6	380.2	380.7	386.0	381.2	382.6	379.8	380.6	379.3	4.45
10	372.9	374.1	377.2	378.8	381.4	381.1	377.5	379.0	382.1	379.1	378.3	3.03
11	374.0	375.9	377.2	382.1	384.1	383.5	384.5	385.1	383.7	382.9	381.3	4.00
12	374.4	376.3	380.7	381.3	384.2	382.5	380.8	384.0	383.2	382.3	381.0	3.22
13	373.5	373.7	378.2	381.3	384.9	382.5	381.4	379.5	380.1	381.3	379.6	3.64
14	373.7	373.0	378.9	382.1	382.6	378.6	381.1	380.3	378.3	379.7	378.8	3.21
15	373.3	367.3	376.6	376.1	376.6	369.2	383.6	370.7	385.6	384.3	376.3	6.46
16	371.9	372.3	376.6	378.0	376.3	378.5	380.8	378.7	377.2	377.0	376.7	2.76
17	371.8	372.6	376.9	379.2	381.2	376.4	378.0	377.8	383.5	376.2	377.4	3.53
18	373.3	365.3	374.2	378.4	379.5	375.0	371.3	377.6	385.0	383.5	376.3	5.82
19	370.6	375.0	376.5	379.3	379.4	384.1	378.5	380.4	379.0	380.3	378.3	3.63
20	372.9	374.2	378.4	380.0	379.8	379.5	380.1	381.4	379.2	379.8	378.5	2.75

Table 4.53: Final capability study for case 4 Rastrigin's function

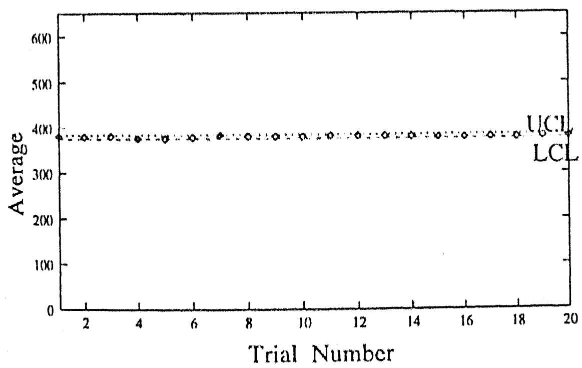

Figure 4.29: Control chart for average of on-line performance after variation reduction case 4 on Rastrigin's function

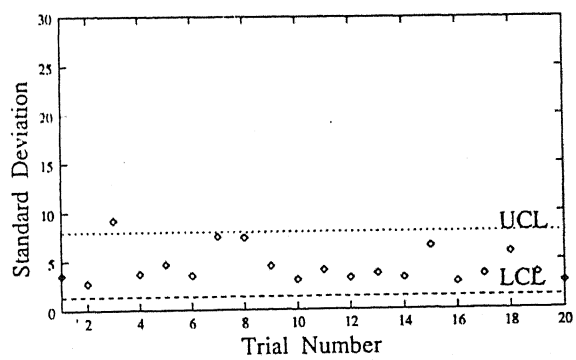

Figure 4.30: Control chart for standard deviation of on-line performance after variation reduction case 4 on Rastrigin's function

4.3.6 Discussion of Result of Rastrigin's Function

In the Table 4.54 the average and the standard deviation of the final capability are shown for each case. The value of PS, CP and MP are given at which we have to fix it with standard deviation of 5, 0.05 and 0.005 respectively.

Table 4.54: Result of Rastrigin's function

		PS	CP	MP	average	std. deviation
CASE 1	Proportionate selection single point crossover	250	0.5	0.1	371.49	6.057
CASE 2	proportionate selection uniform crossover	250	1	0.1	379.61	5.89
CASE 3	tournament selection single point crossover	98	0.5	0.1	409.2	5.845
CASE 4	tournament selection uniform crossover	43	0.66	0.1	378.54	4.78

We see that if our aim is to minimize only on-line performance then we will choose proportionate selection as selection operator and single point crossover as crossover operator (case 1). But if our aim is to minimize only variation in on-line performance then tournament selection with uniform crossover (case 4) will be chosen, but we have seen that one point is outside the control limit, hence this case can not be recommended. The next case in which standard deviation is less is case 3, but in this case average on-line performance is high. The next higher standard deviation is case 2, so for Rastrigin's function proportionate selection with uniform crossover are recommended.

In every case PS, CP and MP which are to be targeted, vary. Since we found that proportionate selection with uniform crossover is better, we will set PS at 250, CP at 1.0 and MP at 0.1 as mean with standard deviation of 5 for PS, 0.05 for CP and 0.005 for MP.

To show that the process capability can not be achieved at the set up of parameters corresponding to minimum value of on-line performance, we are taking one set up from initial

capability study, corresponding to minimum on-line performance. The minimum value is achieved in initial capability study for proportionate selection, uniform crossover, PS = 61, CP = 0.9969, and MP = 0.0022.

Now with mean at PS = 61, CP = 0.9969, and MP = 0.0022 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$ and $\sigma_{MP} = 0.005$ capability study is done. The data are shown in Table 4.55, and the control chart for average is shown in Figure 4.31 and control chart for standard deviation is shown in Figure 4.32. We see that lot of points are going outside the control limits. hence the process is not capable.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	192.5	170.0	162.7	177.5	182.1	194.8	167.5	174.0	164.3	174.6	176.0	11.02
2	134.1	128.6	130.2	128.5	136.9	129.0	140.3	134.0	130.0	125.8	131.7	4.45
3	139.0	123.3	151.9	135.1	152.5	141.8	136.0	164.7	127.4	143.6	141.5	12.39
4	138.6	138.5	124.7	157.4	136.7	150.3	196.9	141.4	136.4	142.5	146.3	19.76
5	123.5	140.8	142.7	154.1	145.2	154.3	140.8	175.3	150.4	152.7	148.0	13.30
6	206.4	198.2	208.9	218.2	185.1	204.3	230.2	191.6	214.2	212.4	207.0	13.13
7	158.3	167.5	166.6	210.5	192.1	161.7	178.2	189.1	222.3	159.6	180.6	22.40
8	139.0	133.7	130.7	132.4	166.9	170.2	145.1	155.7	175.9	154.5	150.4	16.70
9	279.8	240.1	251.4	293.8	260.5	276.6	248.0	273.5	281.4	270.2	267.5	16.99
10	126.7	125.4	135.7	129.4	137.9	136.9	140.6	134.6	134.1	131.2	133.3	4.95
11	275.4	278.7	290.4	292.0	322.8	337.2	295.1	336.0	288.4	316.5	303.3	22.94
12	277.0	292.8	297.5	302.3	286.6	282.2	285.0	268.6	275.2	268.7	283.6	11.56
13	183.1	178.1	145.9	152.0	148.9	152.8	151.3	165.4	158.6	148.1	158.4	13.02
14	155.6	156.7	210.0	219.4	229.4	212.4	230.0	204.3	224.0	178.3	202.0	28.42
15	153.6	162.7	142.7	121.6	145.3	149.2	143.8	183.5	150.2	157.3	151.0	15.84
16	195.7	219.7	215.4	216.2	217.3	185.4	218.1	219.4	197.0	210.9	209.5	12.22
17	178.5	174.0	180.2	194.6	197.7	163.1	182.8	198.1	176.7	214.1	186.0	14.91
18	134.0	133.7	131.3	133.0	157.3	153.3	146.5	154.0	150.6	187.1	148.1	16.99
19	270.8	254.3	268.2	244.1	286.2	268.2	280.4	271.0	249.7	270.6	266.4	13.22
20	121.4	123.2	125.5	127.4	126.1	124.6	129.7	123.5	125.6	134.7	126.2	3.77

Table 4.55: Capability study for checking the process capability at one of minimum point in initial capability study

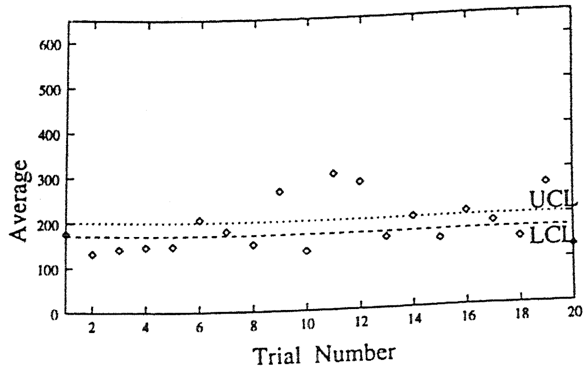

Figure 4.31: Control chart for average of on-line performance

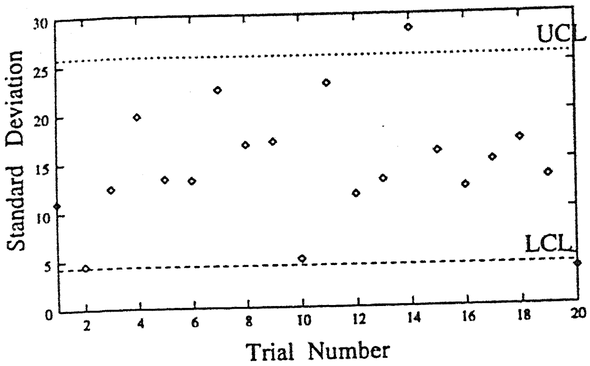

Figure 4.32: Control chart for standard deviation of on-line performance

4.4 Test Function 4: Welded Beam Problem

4.4.1 Initial Capability Study

To check the need for variation reduction capability study is done. The on-line performance measured for last 50 % of the generation are shown in Table 4.56. The average and standard deviation of each group are also shown in Table 4.56.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	380.1	376.1	349.3	333.0	374.0	358.0	364.6	300.2	417.8	381.8	363.5	31.62
2	26.2	24.5	22.1	23.7	22.9	24.3	23.2	22.5	22.6	30.4	24.2	2.49
3	9.1	9.6	9.8	5.3	5.1	10.8	7.0	3.3	3.6	3.9	6.7	2.87
4	21.2	17.2	16.6	18.7	16.9	17.9	23.4	18.7	17.9	16.8	18.5	2.17
5	23.4	20.6	20.9	10.0	14.1	22.3	'17.6	9.2	9.0	14.3	16.1	5.58
6	426.7	408.1	440.5	437.4	444.5	397.8	496.3	441.3	432.1	459.1	438.4	27.02
7	32.4	31.9	31.8	31.6	30.7	29.9	30.6	32.3	31.8	33.0	31.6	0.93
8	18.6	26.3	19.4	20.5	21.1	22.4	17.1	21.2	22.6	22.6	21.2	2.54
9	2179.2	2366.7	2194.6	1972.8	1415.1	2382.5	2359.7	1798.3	1761.8	2035.6	2046.6	316.23
10	1175.2	782.7	948.8	1202.1	1176.3	1037.9	857.7	957.0	987.7	1039.6	1016.5	139.42
11	600.0	470.0	516.2	481.9	556.2	433.2	498.8	474.8	465.0	608.3	510.5	59.27
12	1354.7	1876.6	2043.6	1179.0	1839.0	1900.5	1908.1	1657.7	1707.3	1943.5	1741.0	276.31
13	1745.1	2143.4	1806.0	2187.8	1718.1	2276.7	2047.2	1760.4	1981.2	1814.5	1948.0	205.97
14	35.4	34.8	35.6	34.2	36.5	36.5	35.7	33.7	34.6	35.9	35.3	0.95
15	259.7	264.6	266.2	253.7	272.1	275.0	269.9	254.7	241.5	280.9	263.8	11.69
16	1044.8	1035.7	1008.3	1112.0	981.1	1050.0	976.8	1042.5	945.5	917.9	1011.5	57.23
17	15.9	17.0	17.4	15.0	15.6	16.9	16.9	16.4	15.6	17.2	16.4	0.82
18	9.1	12.5	9.3	8.4	10.8	11.1	7.4	9.9	11.1	7.8	9.7	1.64
19	512.8	487.8	440.6	494.2	458.1	477.7	500.8	487.3	424.9	466.3	475.1	27.57
20	392.1	401.2	460.0	372.2	419.3	423.4	366.4	374.1	430.1	441.1	408.0	31.81

Table 4.56: Initial capability study for welded beam problem

These data were used to construct the average and standard deviation *control charts* shown in Figure 4.33 and Figure 4.34.

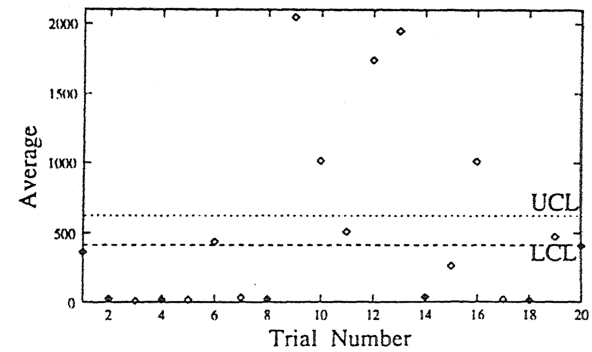

Figure 4.33: Control chart for average on-line performance before variation reduction on welded beam problem

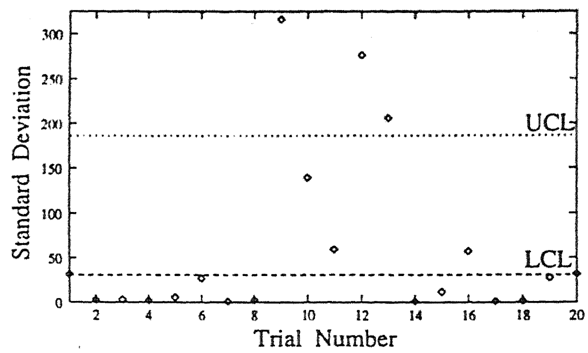

Figure 4.34: Control chart for standard deviation of on-line performance before variation reduction on welded beam problem

We see from Figure 4.33 and Figure 4.34 that lot of points are going outside the control limit. So there is need to reduce the variation. For reducing the variation four cases as described in previous chapter is selected and for each case variation reduction technique is applied.

4.4.2 CASE 1 (Proportionate Selection and Single Point Crossover)

27 trials experiment data are shown in Table 4.57.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	39.5	37.9	40.0	38.3	37.3	38.0	37.2	37.1	39.7	39.6	38.47	1.16
2	100	1.00	0.001	7.5	6.3	7.8	8.0	6.9	7.6	8.1	7.3	7.5	6.3	7.34	0.65
3	10	0.50	0.100	28.6	37.2	32.0	34.8	25.5	31.6	26.4	32.0	27.1	42.1	31.72	5.22
4	100	0.50	0.001	4.3	7.2	8.8	8.4	4.6	6.7	10.0	10.2	7.5	4.7	7.22	2.17
5	10	1.00	0.001	13.9	11.3	13.1	7.1	6.4	7.8	5.3	11.0	11.5	8.0	9.54	2.99
6	10	1.00	0.100	34.7	33.1	29.4	33.8	40.0	44.2	28.8	28.4	29.0	29.6	33.11	5.35
7	100	1.00	0.100	39.8	40.5	41.4	40.9	40.2	40.9	39.9	41.5	41.9	41.6	40.86	0.72
8	10	0.50	0.001	11.6	18.7	12.1	14.5	8.7	4.9	7.5	9.0	7.3	9.5	10.38	3.98
9	55	0.75	0.051	22.2	24.6	23.1	23.7	22.9	24.2	23.7	22.1	24.0	25.0	23.55	0.98
10	55	0.50	0.001	4.3	4.3	4.0	4.1	4.5	4.4	4.1	4.6	5.0	4.4	4.38	0.28
11	55	0.50	0.100	35.3	34.8	38.1	37.1	36.6	36.0	35.9	33.7	36.4	34.6	35.85	1.30
12	55	1.00	0.001	4.0	3.8	3.1	2.8	3.2	3.1	2.7	3.2	3.1	3.1	3.21	0.39
13	55	1.00	0.100	37.7	38.3	35.6	39.6	38.7	40.5	38.3	37.2	38.2	39.4	38.36	1.36
14	10	0.75	0.001	14.5	9.1	11.8	12.1	8.1	6.9	8.5	7.3	10.2	8.3	9.66	2.44
15	10	0.75	0.100	37.9	35.6	38.2	32.8	31.3	29.0	32.8	27.6	29.5	30.0	32.47	3.71
16	100	0.75	0.001	7.7	6.3	8.0	8.9	6.2	6.9	8.2	8.7	9.8	8.8	7.96	1.17
17	100	0.75	0.100	38.2	40.6	41.2	39.1	40.1	40.4	38.8	39.0	39.4	40.0	39.68	0.93
18	10	0.50	0.051	15.3	28.2	23.3	16.9	24.3	17.1	33.8	17.8	18.8	35.1	23.06	7.22
19	10	1.00	0.051	23.9	33.6	17.2	29.6	22.2	18.5	33.2	26.0	25.3	28.1	25.77	5.57
20	100	0.50	0.051	25.2	24.0	24.0	24.1	23.8	25.0	24.4	24.5	23.6	25.0	24.36	0.54
21	100	1.00	0.051	25.1	27.0	25.8	29.1	26.6	29.9	26.3	26.6	27.9	26.6	27.09	1.46
22	55	0.75	0.001	3.9	3.7	3.4	3.7	4.0	4.1	3.3	3.5	3.6	3.6	3.67	0.25
23	55	0.75	0.100	35.2	36.7	37.8	36.8	38.2	37.8	36.6	36.3	37.9	37.0	37.02	0.92
24	10	0.75	0.051	15.4	15.9	19.0	16.5	17.2	19.9	15.7	14.7	18.0	25.9	17.81	3.28
25	100	0.75	0.051	24.0	24.8	24.2	24.5	27.0	25.9	25.1	25.2	26.1	25.5	25.24	0.93
26	55	0.50	0.051	21.8	23.0	24.9	22.7	23.0	21.6	23.8	20.7	22.3	23.1	22.69	1.18
27	55	1.00	0.051	24.4	24.6	25.3	26.0	24.9	25.6	24.0	23.1	25.6	25.8	24.94	0.93

Table 4.57: Data obtained from 27 trials on welded beam problem case 1

The significant effects of the parameters, their interactions and the quadratic effects are shown in Table 4.58 for average and in Table 4.59 for standard deviation.

Design parameter		Effect
MP	mutation prob.	29.352
PS*MP	population size * mutation prob.	4.8
PS	population size	2.74
MP*MP	mutation prob. (quad)	2.11
CP*MP	crossover prob.* mutation prob.	1.36
PS*PS	population size(quad.)	-1.35
CP	crossover prob.	1.34

Table 4.58: Effects table for average case 1

Design parameter		Effect
PS	population size	-3.33
MP*MP	mutation prob.(quad.)	-1.9
PS*MP	population size * mutation prob.	-1.00

Table 4.59: Effects table for standard deviation case 1

Response surface study

Based on 27 trials data in Table 4.57, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 10.522 - 0.09798PS - 0.844CP + 186.95MP + 0.00067PS^2 \\ &\quad -18.4MP^2 + 69.552CP \times MP + 1.0778PS \times MP \qquad (4.34)\end{aligned}$$

The best fit surface through standard deviation is:

$$\log(\sigma) = 1.0272 - 0.01377PS + 53.68MP^2 - 0.0341PS \times MP \qquad (4.35)$$

Analysis by direct observation

- Set PS = 100 and MP = 0.0505 in order to minimize variation.
- Set CP = 0.5 in order to reduce the on-line performance.

Analysis by Variation transmission

Minimization of equation of standard deviation obtained from equation 4.34 gives:

$$\sigma = 1.168 \text{ at } PS = 10,\ CP = 0.818,\ MP = .064.$$

the standard deviation of on-line performance from equation 4.35 will be 2.966, at PS = 10, CP = 0.818 and MP = 0.064 . The standard deviation found from equation 4.35 by direct observation method is 0.68. This means that result from analysis of variation transmission method is not good.

So with mean at PS = 100, CP = 0.5 and MP = 0.0505 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$, and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.4.2.1 Final Capability Study for Case 1

The data are given in Table 4.60. The control chart for average and standard deviation is also given in Figure 4.35 and Figure 4.36 . We see that all points are within the control limits hence the process is capable. Now we try for next case.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	23.9	26.3	25.6	25.4	27.2	28.1	26.1	26.7	25.0	25.2	26.0	1.19
2	23.6	22.6	22.1	23.2	21.2	23.5	22.9	23.6	23.4	23.2	22.9	0.75
3	21.1	21.4	20.5	21.0	22.4	23.1	21.7	21.6	23.2	21.6	21.8	0.89
4	23.4	25.4	25.7	24.0	22.0	24.2	24.9	23.1	23.2	23.2	23.9	1.17
5	21.5	21.0	21.1	20.0	23.4	20.3	21.9	21.6	20.9	23.6	21.5	1.18
6	24.1	24.6	23.6	23.8	24.4	26.8	24.8	24.3	26.7	24.9	24.8	1.10
7	25.0	22.8	23.7	22.9	24.3	25.4	24.3	25.1	25.3	25.3	24.4	0.99
8	22.4	21.5	19.7	23.7	21.8	24.5	22.1	20.8	22.8	22.4	22.2	1.37
9	27.6	28.8	27.5	26.7	27.6	27.6	26.2	27.3	27.3	25.1	27.2	1.00
10	23.5	23.9	26.3	25.3	24.2	24.5	26.9	23.2	23.9	24.6	24.6	1.20
11	26.9	26.3	27.2	30.0	27.6	27.8	27.9	30.1	27.9	28.4	28.0	1.22
12	24.4	25.7	27.7	27.0	28.3	29.9	28.3	26.5	28.9	26.5	27.3	1.63
13	24.0	26.3	25.5	24.9	25.3	23.8	27.6	24.9	24.0	27.5	25.4	1.38
14	22.4	24.4	22.7	21.8	24.2	23.0	23.1	23.2	24.2	25.5	23.4	1.10
15	21.6	22.3	20.7	21.2	20.5	20.7	21.1	20.7	21.5	20.6	21.1	0.57
16	25.5	24.4	25.2	25.7	24.8	25.5	23.7	27.0	25.6	23.8	25.1	0.99
17	24.5	22.7	24.8	25.2	25.7	23.1	25.7	25.4	26.0	24.7	24.8	1.09
18	22.5	22.2	23.2	23.4	21.2	20.6	23.5	21.2	21.0	21.4	22.0	1.08
19	27.0	25.9	25.1	27.4	27.1	27.8	27.9	28.1	28.2	27.9	27.2	1.02
20	23.6	24.7	24.6	25.9	23.0	24.2	23.9	23.7	23.0	23.1	24.0	0.92

Table 4.60: Final capability study for case 1 welded beam problem

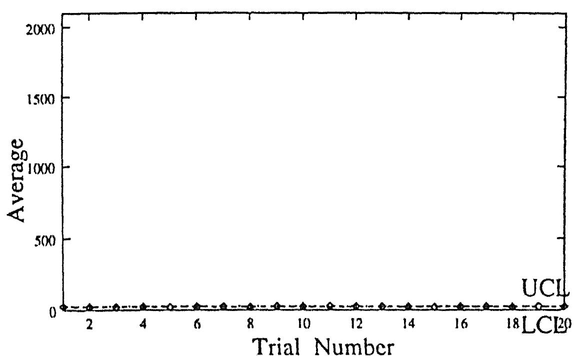

Figure 4.35: Control chart for average of on-line performance after variation reduction for case 1 on welded beam problem

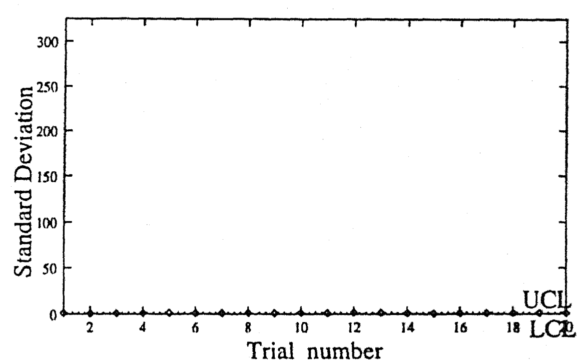

Figure 4.36: Control chart for standard deviation of on-line performance after variation reduction for case 1 on welded beam problem

4.4.3 CASE 2 (Proportionate Selection and Uniform Crossover)

27 trial experiment data are shown in Table 4.61.

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	42.1	40.9	41.4	39.5	41.5	41.5	40.3	38.9	42.8	41.3	41.00	1.18
2	100	1.00	0.001	3.5	6.6	8.2	5.0	8.0	4.6	9.8	4.9	6.2	5.7	6.28	1.92
3	10	0.50	0.100	36.5	27.1	30.3	33.8	27.5	44.6	26.8	28.3	26.3	29.7	31.10	5.77
4	100	0.50	0.001	5.7	5.6	5.3	3.7	4.9	8.0	6.4	4.7	6.3	6.3	5.70	1.16
5	10	1.00	0.001	11.1	18.5	22.7	7.2	5.6	7.4	8.2	7.4	6.9	16.6	11.16	5.94
6	10	1.00	0.100	29.3	32.1	28.3	27.3	27.2	51.7	32.6	28.5	28.9	31.1	31.70	7.30
7	100	1.00	0.100	43.6	45.0	45.6	43.6	41.8	42.6	45.9	44.8	47.7	43.6	44.43	1.73
8	10	0.50	0.001	7.5	8.5	7.9	8.1	8.9	5.9	8.1	9.4	13.3	5.3	8.29	2.17
9	55	0.75	0.051	25.0	25.8	22.6	25.2	26.3	26.0	25.7	23.3	22.6	24.1	24.66	1.40
10	55	0.50	0.001	3.8	3.2	3.0	3.1	3.1	3.1	3.4	3.1	3.0	3.0	3.16	0.24
11	55	0.50	0.100	39.9	38.2	39.3	37.5	36.9	37.8	39.6	36.1	38.4	38.1	38.19	1.20
12	55	1.00	0.001	5.2	3.2	3.0	3.1	2.9	3.3	3.2	3.0	3.3	3.1	3.35	0.66
13	55	1.00	0.100	40.4	39.9	40.9	39.7	40.9	42.7	40.6	39.8	40.2	42.0	40.70	0.97
14	10	0.75	0.001	11.2	15.8	12.8	9.2	4.6	9.7	8.6	7.8	7.7	16.6	10.40	3.76
15	10	0.75	0.100	86.7	29.9	26.9	26.6	33.8	29.4	34.1	30.1	24.6	35.1	35.74	18.25
16	100	0.75	0.001	5.4	7.2	8.0	7.8	7.0	7.8	7.2	11.4	10.1	4.3	7.61	2.05
17	100	0.75	0.100	43.5	41.2	43.9	44.4	40.7	42.1	44.2	42.7	42.3	42.2	42.72	1.25
18	10	0.50	0.051	17.1	23.7	32.8	23.6	21.1	30.9	16.4	16.3	26.6	18.8	22.72	5.93
19	10	1.00	0.051	24.3	23.6	22.7	26.7	17.5	24.7	25.6	20.0	17.2	17.1	21.94	3.69
20	100	0.50	0.051	25.8	28.7	25.1	24.8	25.6	26.8	28.5	24.1	26.1	26.4	26.19	1.49
21	100	1.00	0.051	27.7	27.1	29.9	27.6	27.7	29.8	30.1	27.9	29.4	28.9	28.60	1.12
22	55	0.75	0.001	3.0	3.0	3.1	3.3	5.5	3.1	3.1	10.7	2.9	3.2	4.08	2.46
23	55	0.75	0.100	38.8	37.4	39.9	40.5	40.5	40.1	37.5	38.7	39.8	39.1	39.24	1.13
24	10	0.75	0.051	31.4	21.7	21.2	16.1	17.6	30.5	18.8	17.3	15.5	19.6	20.98	5.62
25	100	0.75	0.051	26.5	27.3	28.6	26.3	26.6	26.7	28.2	28.3	26.2	26.0	27.06	0.96
26	55	0.50	0.051	21.9	23.3	24.7	23.2	24.1	25.6	21.9	21.7	24.7	26.3	23.74	1.62
27	55	1.00	0.051	24.6	26.0	25.2	26.8	25.5	27.5	24.1	23.9	26.7	26.1	25.64	1.20

Table 4.61: Data obtained from 27 trials on welded beam problem case 2

The significant effects of the parameters, their interactions and the quadratic effects are shown in Table 4.62 for average and in Table 4.63 for standard deviation.

Design parameter		Effect
MP	mutation prob.	31.65
PS*MP	population size * mutation prob.	6.65
PS	population size	3.95
MP*MP	mutation prob. (quad)	2.12
CP	crossover prob.	1.52
PS*PS	population size (quad)	-1.004

Table 4.62: Effects table for average case 2

Design parameter		Effect
PS	population size	-5.06
PS*MP	population size * mutation prob.	-3.4
PS*PS	population size(quad.)	-2.75
MP	mutation prob.	2.04
CP*CP	crossover prob.(quad.)	1.581

Table 4.63: Effects table for standard deviation case 2

Response surface study

Based on 27 trials data in Table 4.61, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 7.179 - 0.083PS + 3.167CP + 232.29MP + 0.000482PS^2 \\ &\quad +72.2MP^2 + 1.469PS \times MP \qquad (4.36)\end{aligned}$$

The best fit surface through standard deviation is:

$$\begin{aligned}\log(\sigma) &= 1.6589 - 0.062PS + 10.81MP + 0.000485PS^2 + 0.2289CP^2 \\ &\quad -0.1269PS \times MP \qquad (4.37)\end{aligned}$$

Analysis by direct observation

- Set PS = 62 , CP = 0.5 and MP = 0.001 in order to minimize variation.

Analysis by Variation transmission

Minimization of equation of standard deviation obtained from equation 4.36 gives :

$$\sigma = 1.275 \text{ at } PS = 10,\ MP = 0.033$$

Now putting these values in the equation 4.36 , and minimizing that will give value of CP at which we have to target it.

minimum on-line performance = 16.2 at CP = 0.5.

standard deviation from equation 4.37 will be 4.3 at PS=10 , CP =0.5 and MP = 0.033.

From direct observation method standard devaition of on-line performance = 0.77. We can see that result from both the method doesn't match.

So with mean at PS = 62, CP = 0.5 and MP = 0.001 and standard deviation σ_{PS} = 5, σ_{CP} = 0.05, and σ_{MP} = 0.005 final capability study is done to verify the expected improvement.

4.4.3.1 Final Capability Study for Case 2

The data are given in Table 4.64. The control chart for average and standard deviation is also given in Figure 4.37 and Figure 4.38 . We see that all points are within the control limits hence the process is capable. Now we try for next case.

Table 4.64: Final capability study for case 2 welded beam problem

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	4.4	18.3	4.0	7.5	6.2	4.2	6.3	8.9	10.9	11.6	8.2	4.44
2	3.6	3.4	3.1	3.4	3.4	3.5	3.3	5.5	4.2	3.2	3.6	0.70
3	3.8	3.2	8.8	3.2	3.1	3.0	4.1	3.0	2.9	2.7	3.8	1.81
4	4.1	3.6	3.6	4.1	4.3	3.9	2.9	3.0	2.7	2.7	3.5	0.62
5	4.2	3.0	2.7	2.8	2.7	2.9	2.7	2.7	2.8	2.9	2.9	0.45
6	4.3	5.4	5.4	4.6	5.0	5.1	5.5	5.2	4.7	5.5	5.1	0.39
7	4.4	4.5	4.5	5.0	4.0	4.2	5.0	3.9	4.6	5.2	4.5	0.43
8	4.2	2.7	6.8	6.2	9.7	2.7	3.4	5.3	8.8	3.2	5.3	2.53
9	6.9	9.2	7.6	7.2	7.5	7.4	7.3	7.7	7.7	7.7	7.6	0.60
10	3.5	4.1	4.6	6.3	7.7	7.1	4.2	9.8	7.4	7.3	6.2	2.05
11	7.4	12.2	11.8	8.0	9.9	8.5	14.0	11.1	8.7	8.8	10.0	2.15
12	6.8	9.0	11.0	7.2	6.8	16.9	10.8	8.5	9.4	10.1	9.6	2.99
13	4.1	5.0	7.3	5.1	5.4	3.8	5.0	9.5	5.0	8.0	5.8	1.83
14	7.9	12.7	4.1	6.7	3.5	3.4	3.2	5.4	6.4	3.4	5.7	2.96
15	4.2	3.0	2.8	2.7	2.8	2.8	2.8	2.9	2.8	2.9	3.0	0.44
16	5.0	5.0	5.9	5.0	5.7	4.7	5.8	5.4	4.7	5.5	5.3	0.44
17	3.9	4.3	6.4	4.3	3.7	4.2	3.9	3.7	3.6	4.1	4.2	0.79
18	3.6	3.1	9.8	7.7	9.9	2.6	8.5	9.9	5.3	5.0	6.5	2.95
19	9.4	8.0	8.2	6.5	8.9	9.1	6.7	10.2	8.9	8.0	8.4	1.15
20	10.7	4.1	4.5	6.7	5.0	6.6	3.7	6.2	6.5	3.6	5.8	2.13

4.4.4 CASE 3 (Tournament Selection and Single Point Crossover)

27 trials experiment data are shown in Table 4.65..

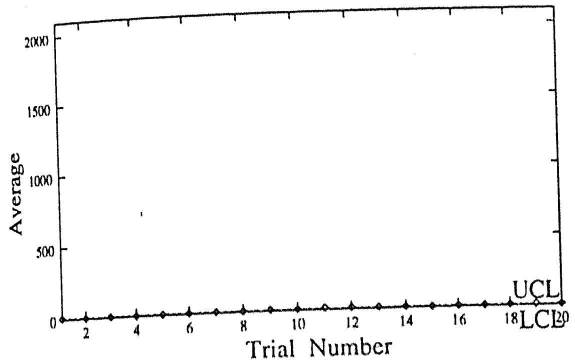

Figure 4.37: Control chart for average of on-line performance after variation reduction case 2 on welded beam problem

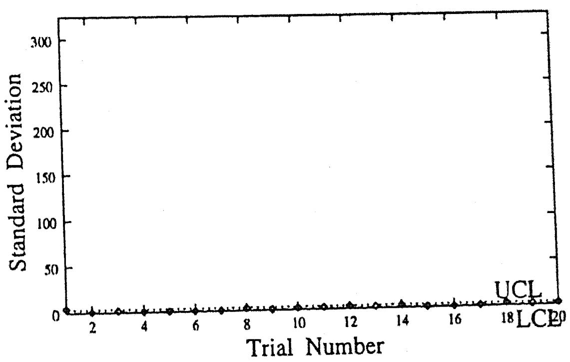

Figure 4.38: Control chart for standard deviation of on-line performance after variation reduction case 2 on welded beam problem

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	456.4	449.2	410.6	392.7	390.1	433.9	456.7	423.5	394.9	498.0	430.61	35.05
2	100	1.00	0.001	2948.7	3541.8	3109.0	2850.6	3403.8	3162.3	3478.3	3403.0	3360.9	3383.6	3264.18	233.42
3	10	0.50	0.100	135.1	128.0	129.9	137.2	131.3	138.1	137.4	140.3	130.4	138.8	134.66	4.37
4	100	0.50	0.001	2948.5	3160.4	3090.8	2993.1	3147.8	3162.0	3253.9	3443.2	3361.1	2887.8	3144.85	176.02
5	10	1.00	0.001	887.0	318.3	304.2	301.6	437.0	492.5	306.1	301.3	371.6	354.6	407.41	180.60
6	10	1.00	0.100	136.3	130.0	135.7	131.2	127.5	134.0	132.8	139.4	139.6	129.7	133.63	4.15
7	100	1.00	0.100	454.3	433.8	420.3	418.6	433.4	410.0	386.1	409.3	438.9	411.5	421.61	19.19
8	10	0.50	0.001	1737.6	535.2	335.3	477.0	433.7	492.5	315.9	321.9	371.6	354.5	537.52	428.73
9	55	0.75	0.051	1025.9	865.9	946.7	965.1	578.2	944.8	683.8	749.9	812.1	782.2	835.48	140.73
10	55	0.50	0.001	1595.8	1490.8	1579.5	1563.6	1337.2	1386.6	1595.4	1598.3	1309.7	1284.0	1474.09	130.77
11	55	0.50	0.100	307.1	333.3	330.8	292.5	296.1	309.5	308.7	393.6	369.7	324.2	326.54	32.49
12	55	1.00	0.001	1595.5	1490.5	1579.8	1564.0	1341.8	1376.9	1594.4	1597.9	1309.5	1282.0	1473.23	131.22
13	55	1.00	0.100	311.0	287.1	317.1	353.8	300.0	302.5	322.9	353.7	295.3	316.1	315.95	22.08
14	10	0.75	0.001	1707.0	535.2	374.1	477.1	433.7	492.4	207.0	321.2	371.7	354.6	536.40	418.53
15	10	0.75	0.100	130.2	143.0	141.9	135.3	129.2	130.4	130.6	137.1	134.2	136.9	134.95	5.04
16	100	0.75	0.001	2947.8	3808.2	3129.9	3514.6	3218.3	3135.6	3421.3	3443.9	3359.9	3383.6	3336.30	241.25
17	100	0.75	0.100	401.4	300.9	433.5	453.9	427.7	420.0	400.2	448.8	439.3	438.4	426.32	20.21
18	10	0.50	0.051	201.7	184.1	192.1	206.9	196.2	194.7	193.4	200.9	199.5	177.5	194.69	8.67
19	10	1.00	0.051	194.2	176.5	198.7	172.7	196.8	181.9	215.6	189.8	196.2	196.6	191.91	12.41
20	100	0.50	0.051	1410.2	1506.1	1465.0	1389.3	1420.5	1321.1	1303.7	1434.2	1125.7	1562.9	1393.86	122.10
21	100	1.00	0.051	1382.6	1347.5	1372.2	1404.4	1263.1	1393.2	1367.7	1368.8	1415.7	1386.3	1370.14	42.39
22	55	0.75	0.001	1505.0	1490.8	1579.4	1565.4	1330.8	1386.7	1595.7	1598.3	1309.7	1290.2	1474.90	130.00
23	55	0.75	0.100	280.9	345.0	293.9	344.0	346.6	208.1	379.1	303.1	287.7	289.9	316.82	33.70
24	10	0.75	0.051	204.5	208.3	184.0	194.0	202.3	178.7	186.8	214.2	191.3	178.3	194.24	12.61
25	100	0.75	0.051	1193.9	1273.9	1449.2	1465.5	1476.7	1319.7	1435.1	1458.7	1447.4	1305.9	1382.61	100.09
26	55	0.50	0.051	891.7	632.8	963.4	686.1	746.7	838.9	678.6	648.5	586.1	579.3	725.23	131.90
27	55	1.00	0.051	929.4	661.4	629.6	715.4	605.2	710.4	892.2	918.3	839.7	583.2	748.48	134.43

Table 4.65: Data obtained from 27 trials on welded beam problem case 3

The significant effects of the parameters, their interactions and the quadratic effects are shown in Table 4.66 for average and in Table 4.67 for standard devi ation.

Design parameter		Effect
MP	mutation prob.	-1445.31
PS	population size	1411.67
PS*MP	population size * mutation prob.	-1231.45
MP*MP	mutation prob. (quad)	-234.26
PS*PS	population size (quad)	-125.246
PS*CP	population size * crossover prob.	36.756
CP*CP	crossover prob. (quad.)	32.63

Table 4.66: Effects table for average case 3

Design parameter		Effect
MP	mutation prob.	-210.4
PS*MP	population size * mutation prob.	73.01
MP*MP	mutation prob. (quad)	-46.488
PS*CP	population size * crossover prob.	34.4
CP	crossover prob.	-32.178
CP*MP	crossover prob. * mutation prob.	27.4
CP*CP	crossover prob.(quad.)	19.65
PS*PS	population size (quad.)	-16.05
PS	population size	-9.49

Table 4.67: Effects table for standard deviation case 3

Response surface study

Based on 27 trials data in Table 4.65, the best surface through the average is fitted. The resulting equation is :

$$\begin{aligned}\text{on-line performance} &= 231.016 + 21.3PS - 4415.04MP + 0.0639PS^2 - 88.35CP^2 \\ &\quad +54669.87MP^2 + 2.129PS \times CP - 282.48PS \times MP \quad (4.38)\end{aligned}$$

The best fit surface through standard deviation is:

$$\log(\sigma) = 8.588 + 0.034PS - 8.796CP - 52.693MP - 0.0003PS^2 + 4.668CP^2$$

$$-90.59MP^2 - 0.0031PS \times CP + 27.428CP \times MP$$
$$+0.2531PS \times MP \qquad (4.39)$$

Analysis by direct observation

- Set PS = 10 , CP = 0.75 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of equation of standard deviation obtained from equation 4.38 gives :

$$\sigma = 8.075 \text{ at } PS = 19 \text{ ,CP} = 0.5 \text{ and } MP = 0.088$$

Putting the value PS = 19, CP = 0.5 and MP = 0.088 in the equation 4.39 of standard deviation of on-line performance we get $\sigma = 8.77$. The standard deviation found from equation 4.39 by direct observation method is 2.85. This means that the result from variation transmission method is not good.

So with mean at PS = 10, CP = 0.75, and MP = 0.1 and standard deviation $\sigma_{PS} = 5$, $\sigma_{CP} = 0.05$, and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.4.4.1 Final Capability Study for Case 3

The data are given in Table 4.68. The control chart of average and standard deviation is also given in Figure 4.39 and Figure 4.40. We see that all points are within the control limits exept two points which are slightly above UCL in Figure 4.40 hence the process is capable. Now we try for next case.

4.4.5 CASE 4 (Tournament Selection and Uniform Crossover)

27 trials experiment data are shown in Table 4.69.

Trial number	Run number										average	std. dev.
	1	2	3	4	5	6	7	8	9	10		
1	149.55	146.51	147.84	151.61	154.60	153.50	157.62	156.84	160.63	152.38	153.11	4.47
2	112.14	106.74	107.90	113.61	112.98	105.61	106.03	104.38	113.29	117.14	109.98	4.35
3	107.47	108.45	126.55	113.90	112.97	113.36	102.17	105.50	109.13	111.86	111.14	6.59
4	88.45	74.97	68.95	63.97	77.03	87.31	95.86	83.53	81.19	63.37	78.46	10.83
5	111.10	106.94	119.66	106.39	111.07	107.00	111.50	104.65	112.23	113.18	110.37	4.37
6	122.48	122.23	107.06	144.25	112.55	120.72	131.84	136.77	100.68	130.05	122.86	13.49
7	85.25	112.40	82.34	87.59	85.47	111.71	82.82	99.71	78.59	104.52	93.04	12.80
8	116.09	104.88	121.34	110.65	117.10	109.29	108.68	103.61	100.02	120.65	111.23	7.33
9	161.98	139.02	146.82	126.62	142.10	148.17	142.71	129.92	145.17	158.04	144.06	10.94
10	106.80	104.06	105.63	110.58	102.91	107.39	113.15	103.80	104.45	100.49	106.82	3.35
11	192.04	197.07	188.24	203.24	201.66	179.99	215.33	199.93	208.80	201.12	198.83	10.06
12	184.34	198.28	187.94	195.06	193.09	188.93	170.78	193.42	188.95	187.59	188.84	7.57
13	128.77	134.42	134.55	135.04	137.35	133.09	133.01	130.11	135.62	138.53	134.05	2.98
14	118.79	111.76	102.89	113.29	114.55	112.54	102.56	113.67	103.24	110.89	110.42	5.60
15	115.23	111.44	105.62	108.44	102.89	121.50	116.27	115.49	102.00	116.00	111.49	6.51
16	127.69	119.19	109.91	139.35	102.01	122.60	139.89	144.40	101.99	123.86	123.09	15.28
17	84.58	109.01	80.50	85.19	89.63	109.02	81.65	99.79	87.26	104.67	93.13	11.33
18	117.86	111.05	105.05	115.66	113.94	112.92	113.66	110.79	107.70	117.24	112.59	4.06
19	144.99	148.32	140.74	131.06	180.59	142.64	125.89	133.99	136.59	161.58	144.64	16.08
20	107.64	115.67	104.76	111.84	114.71	106.45	106.55	102.00	101.29	106.79	107.77	4.90

Table 4.68: Final capability study for case 3 welded beam problem

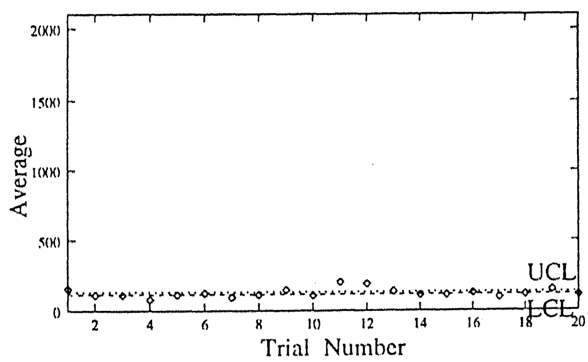

Figure 4.39: Control chart for average of on-line performance after variation reduction case 3 on welded beam problem

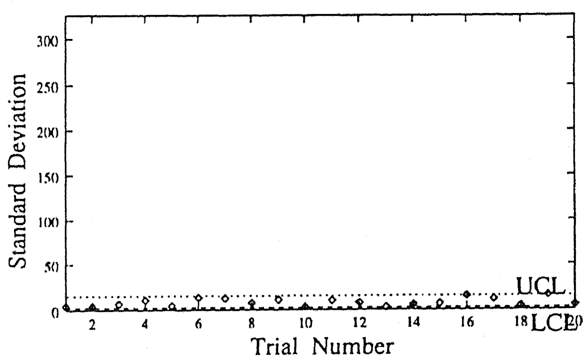

Figure 4.40: Control chart for standard deviation of on-line performance after variation reduction case 3 on welded beam problem

Trial no.	candidate input variables			Run number										on-line performance	
	PS	CP	MP	1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	100	0.50	0.100	437.4	445.9	404.7	446.7	419.5	420.2	423.9	434.6	473.9	404.5	431.14	21.21
2	100	1.00	0.001	2948.5	2961.1	3459.5	3351.1	3106.8	3161.5	3477.5	3066.8	3360.5	3304.5	3249.78	200.38
3	10	0.50	0.100	130.0	137.6	134.0	130.4	144.1	134.3	135.0	141.6	140.0	140.5	136.75	4.78
4	100	0.50	0.001	2948.1	3056.0	3313.3	3700.9	3442.4	3161.8	3478.3	3444.1	3350.2	3360.8	3329.58	220.09
5	10	1.00	0.001	375.1	277.1	314.8	491.9	404.0	320.4	305.9	321.8	371.7	354.0	353.72	61.64
6	10	1.00	0.100	127.2	139.0	145.2	126.5	138.6	128.2	133.8	141.0	130.2	146.3	135.61	7.44
7	100	1.00	0.100	398.6	400.8	400.7	414.9	368.9	430.0	402.1	393.0	425.2	447.0	408.12	21.93
8	10	0.50	0.001	352.6	414.6	531.6	492.7	406.8	492.5	307.1	321.3	371.0	354.5	404.48	77.84
9	55	0.75	0.051	653.7	588.1	730.2	722.7	938.2	832.6	908.5	686.0	866.8	602.2	752.90	126.27
10	55	0.50	0.001	1595.1	1490.2	1579.3	1564.4	1337.3	1372.2	1595.7	1598.1	1304.1	1290.4	1472.70	131.65
11	55	0.50	0.100	303.7	283.2	325.1	329.1	298.1	315.2	341.0	319.6	290.8	279.0	308.47	20.76
12	55	1.00	0.001	1595.6	1490.5	1579.5	1564.7	1336.5	1380.9	1595.0	1050.0	1309.6	1290.3	1419.25	178.65
13	55	1.00	0.100	313.4	304.7	325.3	303.3	278.5	373.8	287.1	311.0	378.9	360.8	323.68	35.54
14	10	0.75	0.001	352.4	346.3	274.4	492.7	441.9	492.4	327.7	321.9	329.8	348.3	372.78	75.54
15	10	0.75	0.100	127.6	137.1	138.4	140.9	130.1	144.4	134.2	133.1	126.3	136.0	134.82	5.76
16	100	0.75	0.001	2948.2	3347.3	2966.8	3506.0	3459.5	3161.7	3173.9	3443.2	3359.8	3383.4	3274.98	201.31
17	100	0.75	0.100	400.1	436.7	405.4	404.7	410.7	378.5	456.9	428.0	412.5	474.7	420.82	28.67
18	10	0.50	0.051	202.2	207.0	192.3	200.3	191.8	190.1	213.4	198.3	193.7	177.0	196.62	10.09
19	10	1.00	0.051	183.1	182.1	192.8	199.6	198.2	190.3	180.3	191.6	187.2	194.0	189.93	6.64
20	100	0.50	0.051	1459.2	1347.3	1371.9	1269.1	1347.4	1404.7	1416.6	1332.1	1414.0	1358.3	1372.05	53.75
21	100	1.00	0.051	1242.7	1438.9	1420.5	1244.7	1285.6	1235.6	1237.5	1505.7	1301.3	1307.8	1322.03	97.71
22	55	0.75	0.001	1591.5	1490.7	1580.1	1565.6	1342.4	1386.7	1595.7	1597.8	1309.6	1289.4	1474.95	129.07
23	55	0.75	0.100	305.5	339.8	317.5	350.4	294.0	294.3	305.5	305.8	312.7	383.0	320.85	28.48
24	10	0.75	0.051	200.7	177.0	204.7	195.4	189.3	207.8	194.8	184.5	192.1	179.2	192.55	10.31
25	100	0.75	0.051	1162.5	1315.5	1314.2	1425.6	1442.3	1525.6	1263.2	1376.2	1435.4	1251.9	1351.25	109.46
26	55	0.50	0.051	1034.5	931.9	917.9	679.1	625.0	656.9	643.8	715.5	830.5	767.9	780.29	142.07
27	55	1.00	0.051	720.0	674.9	650.4	602.4	847.6	661.1	625.8	647.1	838.0	848.9	711.63	96.89

Table 4.69: Data obtained from 27 trials on welded beam problem case 4

The significant effects of the parameters, their interaction and the quadratic effects are shown in Table 4.70 for average and in Table 4.71 for standard deviation.

Design parameter		Effect
PS	population size	1449.16
MP	mutation prob.	-1414.66
PS*MP	population size * mutation prob.	-1311.74
MP*MP	mutation prob. (quad)	-235.22
PS*PS	population size (quad)	-119.3
CP	crossover prob.	-35.37
CP*MP	crossover prob. * mutation prob.	29.176
PS*CP	population size * crossover prob.	-15.7

Table 4.70: Effects table for average case 4

Response surface study

Based on 27 trials data in Table 4.69, the best surface through the average is fitted. The resulting equation is :

$$\text{on-line performance} = 34.343 + 24.98PS + 43.33CP - 2302.19MP + 0.06PS^2$$

Design parameter		Effect
MP	mutation prob.	-122.4
PS	population size	77.164
PS*MP	population size * mutation prob.	-58.82
PS*PS	population size (quad)	31.345
MP*MP	mutation prob. (quad)	-8.01
PS*CP	population size * crossover prob.	6.994

Table 4.71: Effects table for standard deviation case 4

$$+60357.13MP^2 - 0.48PS \times CP - 1748.4CP \times MP$$
$$-301.38PS \times MP \quad (4.40)$$

The best fit surface through standard deviation is:

$$\log(\sigma) = 3.319 + 0.0576PS - 19.287MP - 0.000414PS^2 - 46.11MP^2$$
$$+0.0039PS \times CP + 0.045PS \times MP \quad (4.41)$$

Analysis by direct observation

- Set PS = 10 , CP = 0.5 and MP = 0.1 in order to minimize variation.

Analysis by Variation transmission

Minimization of equation of standard deviation obtained from equation 4.40 gives :

$\sigma = 10.65$ at $PS = 20$, CP = 1.0, and $MP = 0.088$.

Putting the value PS = 20, CP =1 and MP = 0.088 in the equation 4.41 of standard deviation of on-line performance we get $\sigma = 11.11$. The standard deviation found from equation 4.41 by direct observation method is 4.615. This means that result from variation transmission method is not good.

So with mean at PS = 10, CP = 0.5, and MP = 0.1 and standard deviation $\sigma_{PS} =$

5,$\sigma_{CP} = 0.05$ and $\sigma_{MP} = 0.005$ final capability study is done to verify the expected improvement.

4.4.5.1 Final Capability Study for Case 4

The data are given in Table 4.72. The control chart for average and standard deviation is also given in Figure 4.41 and Figure 4.42. We see that all points are within the control limits except some points which are slightly above UCL hence the process is capable.

Trial number	Run number 1	2	3	4	5	6	7	8	9	10	average	std. dev.
1	158.19	145.41	162.45	155.46	157.73	152.35	150.40	146.47	156.68	152.75	153.79	5.37
2	113.76	105.90	107.73	111.10	116.56	113.03	115.22	110.88	117.41	119.53	113.11	4.30
3	111.77	101.19	110.50	114.30	112.05	113.48	110.84	111.47	110.65	103.25	109.95	4.28
4	85.36	147.08	94.85	146.78	63.07	118.74	130.73	137.07	139.48	139.86	120.30	29.26
5	115.27	110.05	111.16	107.08	108.86	112.49	119.24	113.59	108.39	115.19	112.13	3.76
6	132.67	127.80	118.77	123.21	103.12	138.40	145.03	107.91	106.56	141.67	124.51	15.19
7	101.60	145.69	79.36	113.46	105.11	108.31	108.81	95.99	76.92	144.10	107.93	22.93
8	113.22	121.27	110.84	102.03	108.48	115.17	109.74	110.09	109.33	120.70	112.09	5.79
9	156.25	149.20	148.91	138.40	142.47	148.63	129.92	126.48	144.51	161.05	144.58	10.80
10	109.33	106.72	112.52	111.39	113.80	118.68	108.61	109.82	103.17	105.89	109.99	4.41
11	205.14	193.30	209.74	193.22	193.99	207.46	205.24	187.45	206.05	207.35	200.89	7.97
12	186.82	184.05	179.08	177.73	190.13	188.97	187.81	179.28	177.61	173.82	182.53	5.71
13	129.89	139.65	140.04	132.79	139.62	137.72	136.10	131.66	137.36	133.64	135.85	3.64
14	110.46	117.79	107.76	113.99	112.76	116.41	114.23	114.90	114.66	111.47	113.44	2.94
15	112.66	112.12	102.32	105.20	113.17	121.60	115.43	108.55	113.31	114.72	111.91	5.44
16	132.58	132.98	120.91	137.76	105.73	138.03	146.34	172.91	191.33	119.45	139.80	25.46
17	114.80	114.87	76.71	174.01	118.12	160.56	162.04	107.48	79.93	91.29	119.98	34.72
18	115.64	116.37	112.91	115.69	111.63	111.10	116.85	105.68	113.16	123.83	114.29	4.72
19	148.79	148.07	152.66	131.67	144.41	141.67	134.71	132.78	161.13	165.36	146.12	11.49
20	109.58	103.47	112.16	111.50	105.53	104.70	114.35	113.64	106.81	117.86	109.96	4.74

Table 4.72: Final capability study for case 4 welded beam problem

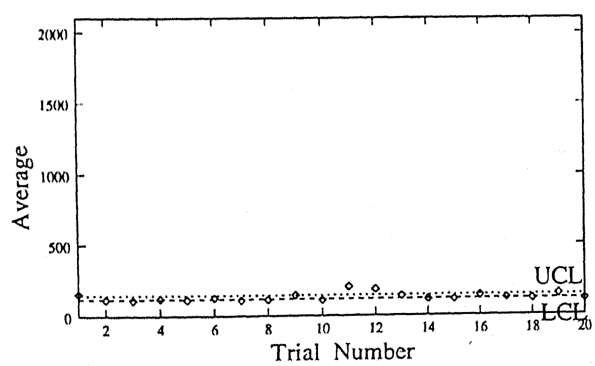

Figure 4.41: Control chart for average of on-line performance after variation reduction case 4 on welded beam problem

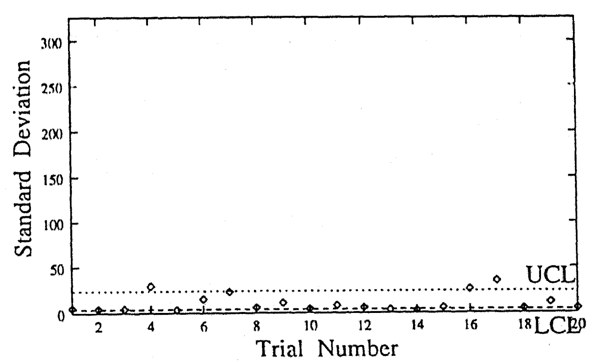

Figure 4.42: Control chart for standard deviation of on-line performance after variation reduction case 4 on welded beam problem

4.4.6 Discussion of Result of Welded Beam Problem

The Table 4.73 represents the average and standard deviation of the capability study after optimization and variation reduction. The value of PS , CP and MP are given at which we have to fix it with standard deviation of 5 , 0.05 and 0.005 respectively.

Table 4.73: Result of welded beam problem

		PS	CP	MP	average	std. deviation
CASE 1	Proportionate selection single point crossover	100	0.5	0.0505	24.381	1.114
CASE 2	proportionate selection uniform crossover	62	0.5	0.001	5.753	1.956
CASE 3	tournament selection single point crossover	10	0.75	0.1	123.29	9.086
CASE 4	tournament selection uniform crossover	10	0.5	0.1	129.16	14.211

We see that if our aim is to minimize only on-line performance then we will choose proportionate selection as selection operator and uniform crossover as crossover operator (case 2). But if our aim is to minimize only variation in on-line performance then proportionate selection with single point crossover (case 1) will be chosen. But in this case on-line performance is very high. So we will choose the case in which variation in on-line performance is less and also on-line performance is less. So for welded beam problem proportionate selection with uniform crossover will give better result.

In every case PS, CP and MP which are to be targeted, vary. Since we found that proportionate selection with uniform crossover is better, we will set PS at 62, CP at 0.5 and MP at 0.001 as mean with standard deviation of 5 for PS, 0.05 for CP and 0.005 for MP.

Chapter 5

Conclusion

In this thesis, a methodology to obtain GA parameters and operators which will produce consistent performance of GAs on a function is designed. The methodology is designed based on the robust design technique commonly used in product design. Five different GA parameters and operators have been chosen for this study. The result on four functions are summarized in the following table. The average column in the table is the average of 20 trials of mean of on-line performance in 10 runs. The standard deviation is within subgroup standard deviation of final capability study.

		PS	CP	MP	average	std. deviation
Function F1	Proportionate selection uniform crossover	105	0.54	0.0818	24.83	0.414
Function F2	proportionate selection uniform crossover	55	0.5	0.1	295.88	15.23
Rastrigin's function	proportionate selection uniform crossover	250	1.00	0.1	379.61	5.89
Welded beam problem	proportionate selection uniform crossover	62	0.5	0.001	5.753	1.956

We see that in all cases proportionate selection with uniform crossover makes the process capable (producing on-line performance within lower and upper bounds). So for GA to be capable, proportionate selection with uniform crossover is recommended. Since none of the previous approaches mentioned in chapter 1 considered the selection operator

and crossover operator together as the parameter affecting performance of GA, we can not compare our results with any past studies. However, there are a number of aspects of our studies, which agree with that of previous studies.

In all the test functions, we see that higher population size is recommended, which is also suggested by Dė Jong (1975) that increasing population size reduces the stochastic effect of random sampling. Goldberg, Deb and Clark (1992) suggested that the population size should depend on the problem being solved (mainly on the function's nonlinearity). Our results also agree that the most complicated problem (Rastrigin's function) require a large population. The value of CP is recommended higher for test function 3. For other test function a low value of CP is recommended. The value of MP is suggested higher for test function 1, 2 and 3. For test function 4, low value of MP is recommended. Since the optimal values of PS , CP and MP are different for different problems, hence it is not recommended to use a fixed value of these parameters for every problem. However a variation reduction technique as suggested here can be used before applying GA to any arbitrary problem. It is observed that for unimodal, well behaved function (such as De Jong function F1), a large population size with a large mutation probability is recommended, if the minimization of the on-line performance is the goal. For a unimodal function but with flat optimal basin, a smaller population size gives consistent performance of GAs. For a multimodal function (such as F3), a large population size with a large crossover probability is recommended. For any other type of function, a similar study can be performed and best GA parameters and operators can be obtained.

To show how this method is applicable to an engineering design problem, the technique has been applied to a welded beam design problem. When GAs have been run with suggested GA parameters and operators consistent GA performance has been observed.

Since statistical tools such as design of experiment are used to find best parameters for a consistent GA performance, the technique is expected to give best GA parameters. We propose to develop a software implementing robust design technique suggested in this thesis.

Such a software can be used by designers and GA practicers to find the best setting of GA parameters and operators.

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